Name: **No:**

MATH 172 FINAL EXAM (26.05.2011)

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: Duration is 90 minutes to solve 5 questions on 4 pages. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct one.

1-) **C**alculate the following indefinite integral (15 points)

$$\int x^2\left(\frac{e^x-1}{x}\right)dx$$

$$\int x^2\left(\frac{e^x-1}{x}\right)dx=\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}-\int xdx = xe^x-\int e^x dx-\frac{1}{2}x^2+C \qquad \begin{cases} u=x & \Rightarrow \quad du=dx \\ dv=e^x dx & \Rightarrow \quad v=e^x \end{cases}$$

$$=xe^x-e^x-\frac{1}{2}x^2+C$$

2-) If $f(x,y,z)=x^2y+2xyz+z^2y$ then evaluate $\dfrac{\partial f}{\partial r}$ where $\begin{cases} x=4r-5s \\ y=r^3-9 \\ z=5s-2r \end{cases}$

(15 points)

$$\frac{\partial f}{\partial r}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial r}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$$

$$=(2xy+2yz)(4)+(x^2+2xz+z^2)(3r^2)+(2xy+2zy)(-2) \quad \textit{sufficient for the exam}$$

$$=8xy+8yz-4xy-4zy+3r^2(x^2+2xz+z^2)$$

$$=4xy+4yz+3r^2(x^2+2xz+z^2)$$

3-) Evaluate the implicit differentiation for given values (15 points)

$$\ln(z^2) = 3x^2 + 5y \qquad \frac{\partial z}{\partial x} = ? \quad \text{for} \quad x = 5, \quad y = -20 \quad \text{and} \quad z = 1$$

$$\frac{\partial}{\partial x}\left(2\ln z = 3x^2 + 5y\right) \quad \Rightarrow \quad \frac{2}{z}\frac{\partial z}{\partial x} = 6x \quad \Rightarrow \quad \frac{\partial z}{\partial x} = 3xz$$

$$\Rightarrow \quad \left.\frac{\partial z}{\partial x}\right|_{\substack{x=5 \\ y=-20 \\ z=1}} = 3(5)(1) = 15$$

4-) The demand functions of the products **A** and **B** are each a function of the prices of **A** and **B** and are given by

$$q_A = \frac{25\sqrt[3]{p_B}}{\sqrt{p_A}} \qquad \& \qquad q_B = \frac{50\,p_A}{\sqrt[3]{p_B{}^2}}$$

respectively. Find the related marginal-demand functions, and determine whether **A** and **B** are competitive products, complementary products, or neither. (10 points)

$$q_A = 25\,p_B^{1/3}\,p_A^{-1/2} \qquad \& \qquad q_B = 50\,p_A\,p_B^{-2/3}$$

$$\frac{\partial q_A}{\partial p_B} = \frac{25}{3}\,p_B^{-2/3}\,p_A^{-1/2} > 0$$

$$\frac{\partial q_B}{\partial p_A} = 50\,p_B^{-2/3} > 0$$

Competitive products

5-) In this problem, find the critical points of the given function. For each critical points, determine by the second derivative test, whether it corresponds to a relative maximum, to a relative minimum, or to neither, or whether the test gives no information. (15 points)

$$f(x,y) = \frac{x^3}{3} + y^2 - x^2 + 2y$$

$$f_x = x^2 - 2x = x(x-2) = 0 \quad \Rightarrow \quad x = 0 \ \ \& \ \ x = 2$$
$$f_y = 2y + 2 = 0 \quad \Rightarrow \quad y = -1$$

Critical Points: (0, -1) and (2, -1)

$$f_{xx} = 2x - 2 \quad \therefore \quad f_{yy} = 2 \quad \therefore \quad f_{xy} = 0$$
$$D(x,y) = f_{xx} f_{yy} - f_{xy}^2 = 2(2x-2) = 4(x-1)$$

For (0, -1), $D(0,-1) = -4 < 0$ neither minimum nor maximum (a saddle point)

For (2, -1), $D(2,-1) = 4 > 0$ $f_{xx}(2,-1) = 2 > 0$ a minimum

6-) Compute $\left(BA^T - C\right)^T$ matrix, if it exists, given that

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$$

(15 points)

$$\left(BA^T - C\right)^T = AB^T - C^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

7-) Solve this system of linear equation by using matrix reduction (15 points)

$$x\text{-}y+z=2$$
$$x+y+z=0$$
$$x\text{-}z=\text{-}3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & -3 \end{array}\right] \xrightarrow[-R_1+R_3]{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & 0 & -2 \\ 0 & 1 & -2 & -5 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -2 & -5 \end{array}\right]$$

$$\xrightarrow[-R_2+R_3]{R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -2 & -4 \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{-R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Hence the solution is just $x = -1,\ y = -1,\ z = 2$