MATH 172 MIDTERM EXAM 27/03/2014

Name and Surname: ______________ **Student ID # :** ________ **Section #:** ______.

Q1 (20)	Q2 (20)	Q3 (20)	Q4(20)	Q5(20)	Total (100)

ATTENTION: Please show all your work in details. DO NOT USE calculators and cellphones. There are 5 questions on 5 pages. Solve all of them. Duration is 60 minutes.

1- (20 P) Evaluate the following indefinite integrals each of them is 10 Points.

(10 P) – (a) $\int \frac{(1+\ln(x))}{x}\,dx$

Let $u = 1 + ln(x)$, $du = \frac{1}{x}\,dx$. *Hence,*

$$\int \frac{(1+ln(x))}{x}\,dx = \int \frac{du}{u} = ln|u| + C = ln|1 + ln(x)| + C$$

where C (any real number) is constant of integration.

(10 P) – (b) $\int (x + 2x^3)e^{x^4+x^2+5}\,dx$

Let $u = x^4 + x^2 + 5$, $\frac{du}{2} = (x + 2x^3)\,dx$. *Hence,*

$$\int (x+2x^3)e^{x^4+x^2+5}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^4+x^2+5} + C$$

where C (any real number) is constant of integration.

2- (20 P) Evaluate the following indefinite integrals each of them is 10 Points.

(10 P) – (a) $\int \frac{3x+5}{x^2-x-42}dx$

By using partial fraction decompositions, we have

$\frac{3x+5}{x^2-x-42} = \frac{3x+5}{(x-7)(x+6)} = \frac{A}{(x-7)} + \frac{B}{(x+6)}$. *Then,* $3x+5 = A(x+6) + B(x-7)$.

For $x = -6$, $-13 = -13B$. $B=1$.

For $x = 7$, $26 = 13A$. $A=2$.

$$\int \frac{3x+5}{x^2-x-42}dx = \int \left(\frac{2}{(x-7)} + \frac{1}{(x+6)}\right)dx$$

$$= 2\ln|x-7| + \ln|x+6| + C$$

$$= \ln\left[(x-7)^2|x+6|\right] + C$$

where C is constant of integration.

(10 P) – (b) $\int (x^2+1)\ln(x)dx$

By using integration by parts, we choose

$u = \ln(x)$, $du = \frac{1}{x}dx$

$dv = (x^2+1)dx$, $v = \frac{x^3}{3} + x$. *Hence,*

$$\int (x^2+1)\ln(x)dx = \left(\frac{x^3}{3}+x\right)\ln(x) - \int \left(\frac{x^3}{3}+x\right)\frac{1}{x}dx$$

$$= \left(\frac{x^3}{3}+x\right)\ln(x) - \int \left(\frac{x^2}{3}+1\right)dx = \left(\frac{x^3}{3}+x\right)\ln(x) - \frac{x^3}{9} - x + C$$

where C is constant of integration.

3- (20 P) Given $y'' = 30x^4 + 20x^3 + 2, y(0) = 1$ and $y'(0) = 1.$ Find $y(x)$.

$$y'(x) = \int (30x^4 + 20x^3 + 2)dx = 6x^5 + 5x^4 + 2x + C,$$

$C = 1$ since $y'(0) = 1. Then\ y'(x) = 6x^5 + 5x^4 + 2x + 1$

$$y(x) = \int (6x^5 + 5x^4 + 2x + 1)dx = x^6 + x^5 + x^2 + x + D$$

$D = 1$ since $y(0) = 1.$ *Hence,*

$$y(x) = x^6 + x^5 + x^2 + x + 1.$$

In fact $y''(x) = 30x^4 + 20x^3 + 2$ *and* $y(0) = 1, y'(0) = 1.$

4- (20 P) Find the area of the region bounded by the parabola $y(x) = x^2 - 5x + 4$ and the line $y(x) = -4x + 4.$

For the limits of integrations, we have $x^2 - 5x + 4 = -4x + 4.$ $x^2 - x = (x)(x - 1) = 0.$ *Then* $x = 0$ *and* $x = 1$ *are the intersection points. The area of the region bounded by the parabola and the line is then*

$$A = \int_0^1 [(-4x + 4) - (x^2 - 5x + 4)]\,dx = \int_0^1 (-x^2 + x)dx$$

$$= -\frac{x^3}{3} + \frac{x^2}{2}\bigg|_0^1 = -\frac{1}{3}\ (1 - 0) + \frac{1}{2}(1 - 0)$$

$$= -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}\ (un)^2.$$

5- (20 P) The demand equation for a product is $p = 0.01q^2 - 1.1q + 30$ and the supply equation is $p = 0.01q^2 + 8$. Determine the consumers' surplus (CS) under the market equilibrium.

In order to find the equilibrium point $0.01q^2 - 1.1q + 30 = 0.01q^2 + 8$. *Then, we have*

$-1.1q + 30 = 8, -1.1q = -22,\ 11q = 220,\ q_0 = 20$ *and for* $p_0 = 0.01\,(20)^2 + 8 = 4 + 8 = 12$. *Hence, the consumers' surplus (CS) is*

$$CS = \int_0^{20} [f(p) - p_0]\, dq = \int_0^{20} \left[0.01q^2 - 1.1q + 30 - 12\right] dq$$

$$= \int_0^{20} \left[0.01q^2 - 1.1q + 18\right] dq = 0.01\frac{q^3}{3} - 1.1\frac{q^2}{2} + 18q\Big|_0^{20}$$

$$= \frac{0.01}{3}(8000) - 1.1\frac{1}{2}(400) + 18(20)$$

$$= \frac{80}{3} - 220 + 360 = \frac{80}{3} + 140 = \frac{500}{3}$$

$$= 166\frac{2}{3}.$$