MATH 172 - PROBLEM SET 4

1. Sketch the given surfaces.
   a) $2x+3y+z=9$ b) $z=x$
2. Find the indicated partial derivatives.
   a) $P=l^3+k^3-lk;\quad \partial P/\partial l,\ \partial P/\partial k$ b) $z=\dfrac{x}{x+y};\quad \dfrac{\partial z}{\partial x},\ \dfrac{\partial z}{\partial y}$
   c) $f(x,y)=\ln\sqrt{x^2+y^2};\quad \dfrac{\partial}{\partial y}[f(x,y)]$ d) $w=e^{x^2yz};\quad w_{xy}=(x,y,z)$
   e) $f(x,y)=xy\ln(xy);\quad f_{xy}(x,y)$ f) $w=e^{x+y+z}\ln xyz;\quad \partial w/\partial y,\ \ \partial^2 w/\partial z\partial x$
3. If $f(x,y,z)=\dfrac{x+y}{xz}$ find $f_{xyz}(2,7,4)$
4. If $f(x,y,z)=(6x+1)e^{y^2\ln(z+1)}$ find $f_{xyz}(0,1,0)$
5. If $w=x^2+2xy+3y^2$, $x=e^r$, and $y=\ln(r+s)$, find $\partial w/\partial r$ and $\partial w/\partial s$
6. If $z^2+\ln(yz)+\ln z+x+z=0$, find $\partial z/\partial y$
7. If a manufacturer's production function is defined by $P=20l^{0.7}k^{0.3}$, determine the marginal productivity functions.
8. A manufacturer's cost for producing *x* units of product X and *y* units of product Y is given by $c=3x+0.05xy+9y+500$ determine the (partial) marginal cost with respect to *x* when *x*=50 and *y*=100.
9. Examine $f(x,y)=x^2+2y^2-2xy-4y+3$ for relative extrema.
10. A company manufactures two products, X and Y, and the joint-cost function for these products is given by $c=0.002(x+y)^2+x+0.25y+8000$, where *c* is the total cost of producing *x* units of X and *y* units of Y. Determine the marginal cost with respect to *x* when *x*=450 and *y*=550.
11. A company's production function is given by $P=40Lk-3L^2-2k^2+500$, where *P* is the total output generated by *L* units of labor and *k* units of capital. Determine: (a) the marginal production function with respect to *L*, (b) the marginal production function with respect to *k*.
12. The demand function for product A is $q_A=\frac{10\sqrt{2p_B}}{p_A}$, and the demand function for product B is $q_B=20+3p_A-2p_B$, where q_A and q_B are the quantities demanded for A and B, respectively, and p_A and p_B are their respective prices. Determine: (a) the marginal demand for A with respect to p_B, (b) the marginal demand for B with respect to p_A, (c) whether A and B are competitive, complementary, or neither.
13. Let $q_A=50-5p_A+6p_B^2$ and $q_B=20\sqrt{p_A}\ p_B^{-1}$ be demand functions, where p_A and p_B are prices for products A and B, respectively. Find all four marginal demand functions.
14. Use implicit partial differentiation to find $\dfrac{\partial z}{\partial y}$ from $e^{xy}+7x^3+8z-19=0$
15. For $\ln(xyz)+e=e^y+1$, the partial derivative $\frac{\partial z}{\partial x}$ evaluated at $x=e^{-2}$, $y=1$, $z=e^3$
16. If $f(x,y)=2x^4y^3-3x^3y^3+4xy-x+2y+4$, find: $f_x(x,y),\ f_y(x,y),\ \ f_{xy}(x,y),\ f_{xy}(-1,1),\ \ f_{yyx}(x,y)$
17. Let $f(x,y,z)=\ln(x^4+6y^2)-2z^4x^2e^{3y}+x^{20}y^3$. Find $\dfrac{\partial^3 f}{\partial x\partial y\partial z}$
18. If $z=(x^2+y^2)^{10}$ where $x=4r^2s^3$ and $y=e^{2r+3s-3}$, then by means of the chain rule, (a) find $\frac{\partial z}{\partial r}$; (b) evaluate $\frac{\partial z}{\partial r}$ when *r*=0 and *s*=1.
19. Determine the critical points of $f(x,y)=2xy-3x-y-x^2-3y^2$ and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information.
20. A television manufacturing company makes two types of TV's. The cost of producing *x* units of type A and *y* units of type B is given by the function $C(x,y)=120+x^3+8y^3-24xy$. How many units of type A and B televisions should the company produce to minimize its cost?