MATH 172 A MIDTERM EXAM (07.04.2009)

Name:
Instructor:
Section:

Q1			Q2		Q3	Q4	Q5		Total

ATTENTION: There are 5 questions on 4 pages. Solve all of them. Duration is 60 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. Evaluate the following integrations. (10 Points each)

(a) $\int \sqrt{x}(5x-3)dx$

Solution:

$$\int \sqrt{x}(5x-3)dx = \int (5x^{\frac{3}{2}} - 3x^{\frac{1}{2}})dx = \frac{2}{\cancel{5}}\cancel{5}x^{\frac{5}{2}} - \frac{2}{\cancel{3}}\cancel{3}x^{\frac{3}{2}} + C = 2x^{\frac{3}{2}}\left(x-1\right) + C$$

(b) $\int \frac{3x^4}{e^{x^5}}dx$

Solution:

$$\int \frac{3x^4}{e^{x^5}}dx = \int \frac{3}{e^u}\frac{du}{5} = \frac{3}{5}\int e^{-u}du = -\frac{3}{5}e^{-x^5} + C \qquad \begin{cases} u = x^5 \\ du = 5x^4 dx \end{cases}$$

(c) $\int \left(\frac{e^{2x} - e}{e^x}\right)dx$

Solution:

$$\int \left(\frac{e^{2x} - e}{e^x}\right)dx = \int \left(e^x - e^{1-x}\right)dx = e^x - (-1)e^{1-x} + C = e^x + e^{1-x} + C$$

2. Find the following integrals (10 Points each)

(a) $\int \frac{\ln x}{x^2} dx$

Solution:

$$\int \underbrace{\ln x}_{u} \underbrace{\frac{dx}{x^2}}_{d\upsilon} = -\frac{1}{x}\ln x - \int -\frac{1}{x}\frac{dx}{x} = -\frac{1}{x}\ln x + \int x^{-2}dx \qquad \begin{cases} u = \ln x & \Rightarrow \quad du = \frac{1}{x}dx \\ d\upsilon = \frac{dx}{x^2} & \Rightarrow \quad \upsilon = -\frac{1}{x} \end{cases}$$

$$= -\frac{1}{x}\ln x - \frac{1}{x} + C = -\frac{1}{x}\left(\ln x + 1\right) + C$$

(b) $\int \frac{x+3}{x^2-1} dx$

Solution:

$$\frac{x+3}{x^2-1} = \frac{x+3}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

$$x+3 = A(x+1) + B(x-1)$$

To find the A and B quantities, we give the special values x=1 and -1:

$$x = +1 \quad \Rightarrow \quad +1+3 = A(1+1) + B(1-1) \quad \Rightarrow \quad A = 2$$

$$x = -1 \quad \Rightarrow \quad -1+3 = A(-1+1) + B(-1-1) \quad \Rightarrow \quad B = -1$$

$$\int \frac{x+3}{x^2-1}dx = \int \frac{2}{x-1}dx + \int \frac{-1}{x+1}dx = 2\ln(x-1) - \ln(x+1) + C = \ln\frac{(x-1)^2}{x+1} + C$$

3. Find the average value of $y = 2 - 3x^2$ function over the [-1, 2] interval. (10 Points)

Solution:

$$\overline{y} = \frac{1}{b-a}\int_a^b y dx = \frac{1}{2-(-1)}\int_{-1}^{2}\left(2-3x^2\right)dx = \frac{1}{3}\left(2x - \cancel{3}\frac{x^3}{\cancel{3}}\right)_{-1}^{2}$$

$$= \frac{1}{3}(4-8) - \frac{1}{3}(-2+1) = -1$$

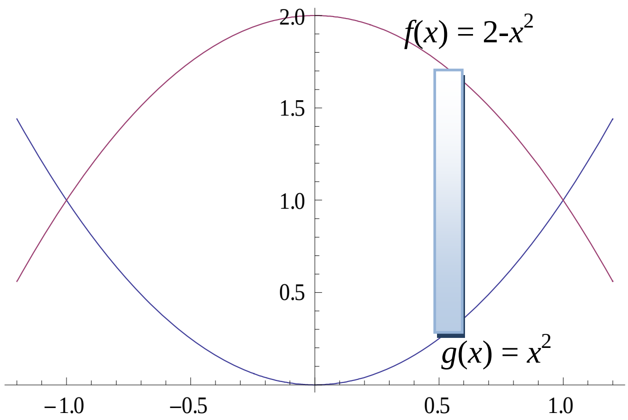
4. Find the area of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x^2$.
(20 Points)

<u>Solution:</u>

The intercept of two functions is given by equating each other:

$$2 - x^2 = x^2 \quad \Rightarrow \quad 2x^2 - 2 = 2(x-1)(x+1) = 0$$

$$\Rightarrow \quad x = 1 \quad \& \quad x = -1$$



The area between two functions is given by

$$Area = \int_a^b (f(x) - g(x))\,dx = \int_{-1}^{1} \left(2 - x^2 - x^2\right)dx = 2\int_{-1}^{1} \left(1 - x^2\right)dx$$

$$= 2\left(x - \frac{x^3}{3}\right)_{-1}^{1} = 2\left[\left(1 - \frac{1}{3}\right) - \left(-1 - \frac{-1}{3}\right)\right] = \frac{8}{3}$$

5. (a) The marginal cost function for a product is given by

$$\frac{dc}{dq} = 0.09q^2 - 1.2q + 4.5$$

where *c* is the total cost (in TL) of producing *q* kilograms of product per month. Assuming that the fixed cost for the same product is 7700 TL per month, calculate the corresponding cost function. (10 Points)

<u>Solution:</u>

$$c = \int \frac{dc}{dq}dq = \int \left(0.09q^2 - 1.2q + 4.5\right)dq = 0.09\frac{q^3}{3} - 1.2\frac{q^2}{2} + 4.5q + C$$

$$= 0.03q^3 - 0.6q^2 + 4.5q + C$$

When no product ($q = 0$) is produced, the cost function is just given by the fixed cost:

$$q = 0 \quad \Rightarrow \quad c = 7700$$

$$c = \left(0.03q^3 - 0.6q^2 + 4.5q + C\right)_{q=0} = C = 7700$$

Hence the corresponding cost function is

$$c = 0.03q^3 - 0.6q^2 + 4.5q + 7700$$

(b) If the demand equation of a product is $p = 400 - q^2$ and the supply equation of a product is $p = 20q + 100$, find market equilibrium point and consumers' surplus (CS). (10 Points)

<u>Solution:</u>

The market equilibrium is obtained from equating the demand equation and the supply equation:

$$400 - q^2 = 20q + 100 \quad \Rightarrow \quad q^2 + 20q - 300 = (q + 30)(q - 10) = 0$$

~~$q = -30$~~ & $q = 10$

When $q = 10$, then $p = 300$. $(q_0, p_0) = (10, 300)$ is an equilibrium point of the market.

The consumers' surplus is then

$$CS = \int_0^{q_0} [f(q) - p_0]dq = \int_0^{10} \left[400 - q^2 - 300\right]dq = \left(100q - \frac{q^3}{3}\right)_0^{10} = 1000 - \frac{1000}{3} \simeq 666.67 \text{ TL}$$