MATH 172 MIDTERM EXAM (07.07.2011) A

Q1			Q2		Q3	Q4		Total

Name: **No:**

ATTENTION: There are 4 questions on 4 pages. Solve all of them. Duration is 60 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1. Evaluate the following integrals. (10 points each)

(a) $\int y^2 (y - \frac{1}{2}) dy$

Solution:

$$\int y^2 (y - \frac{1}{2}) dy = \int (y^3 - \frac{1}{2} y^2) dy = \frac{y^4}{4} - \frac{1}{2}\frac{y^3}{3} + C = \frac{1}{12} y^3 (3y - 2) + C$$

(b) $\int \frac{5e^x}{1+3e^x} dx$

Solution:

$$\int \frac{5e^x}{1+3e^x} dx = 5\int \frac{1}{u}\frac{du}{3} = \frac{5}{3}\ln|u| + C = \frac{5}{3}\ln\left|1+3e^x\right| + C \qquad \Rightarrow \begin{cases} u = 1+3e^x \\ du = 3e^x dx \end{cases}$$

(c) $\int 2\sqrt{2x-1} dx$

Solution:

$$\int 2\sqrt{2x-1} dx = \int u^{1/2} du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3}(2x-1)^{3/2} + C \qquad \Rightarrow \begin{cases} u = 2x-1 \\ du = 2dx \end{cases}$$

2. Find the following integrals (15 points each)

(a) $\int x^2 \ln x dx$

Solution:

$$\begin{cases} u = \ln x & \Rightarrow \; du = \frac{1}{x}dx \\ dv = x^2 dx & \Rightarrow \; v = \frac{x^3}{3} \end{cases}$$

$$\int x^2 \ln x dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\frac{1}{x}dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} + C$$

$$= \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C$$

(b) $\int \frac{x-3}{2x-x^2}dx$

Solution:

First we have to express the rational function in terms of partial fraction:

$$\frac{x-3}{2x-x^2} = \frac{x-3}{x(2-x)} = \frac{A}{x} + \frac{B}{2-x} = \frac{A(2-x)+Bx}{x(2-x)}$$

$$x-3 = A(2-x)+Bx$$

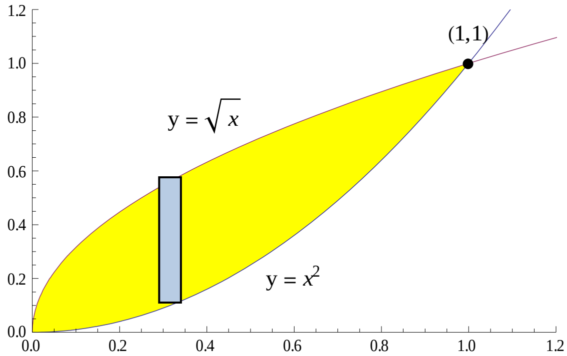
$$x = 0 \quad \Rightarrow \quad A = -3/2$$

$$x = 2 \quad \Rightarrow \quad B = -1/2$$

$$\int \frac{x-3}{2x-x^2}dx = \int \frac{-3/2}{x}dx + \int \frac{-1/2}{2-x}dx = -\frac{3}{2}\ln|x| + \frac{1}{2}\ln|2-x| + C = \frac{1}{2}\ln\left|\frac{2-x}{x^3}\right| + C$$

3. Find the area of the region bounded by the graphs of the given equation: $y = \sqrt{x}$ **and** $y = x^2$**. (20 points)**

Answer:



The curves intersect each other at

$y = x^2 = \sqrt{x} \quad \Rightarrow \quad x\left(x^3 - 1\right) = 0 \quad \Rightarrow \quad x = 0,\, 1 \quad \& \quad y = 0,\, 1$ shown on the figure.

$$area = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \left(\frac{2}{3} x^{\frac{3}{2}} - \frac{x^3}{3}\right)\Bigg|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

4. (a) If the marginal-cost function is given by $dc/dq = q^2 + 7q + 6$ and fixed costs are 2500 TL, determine the total cost of producing six units. *c* is in TL. (10 points)

Solution:

The revenue function, *r*, can be obtained from the marginal-revenue function:

$$c = \int \frac{dc}{dq} dq = \int \left(q^2 + 7q + 6\right) dq = \frac{q^3}{3} + 7\frac{q^2}{2} + 6q + C \text{ .}$$

Here we have to find the *C* constant of integration by assuming that when no units are sold, total cost is 2500 TL as

$$c_{q=0} = 2500 = \left(\frac{q^3}{3} + 7\frac{q^2}{2} + 6q + C\right)_{q=0} = C \text{ .}$$

The total cost producing six units is

$$c = \frac{q^3}{3} + 7\frac{q^2}{2} + 6q + 2500 \Bigg|_{q=6} = 72 + 126 + 36 + 2500 = 2734TL$$

b) The demand equation is $p = 2200 - q^2$ and a supply equation of a product is $p = 400 + q^2$. Determine the producers' surplus (*PS*) under the market equilibrium. (10 points)

Solution:

First we have to find the market equilibrium where demand and supply functions are equal each other as follows:

$$f(q_0) = g(q_0)$$

$$2200 - q_0^2 = 400 + q_0^2 \quad \Rightarrow \quad 2q_0^2 = 1800 \quad \Rightarrow \quad q_0 = 30 \quad \Rightarrow \quad p_0 = 400 + q_0^2 = 1300 \text{ .}$$

Then we can find the producers' surplus (*PS*):

$$PS = \int_0^{30} \underbrace{\left[1300 - 400 - q^2\right]}_{900-q^2} dq = \left(900q - \frac{q^3}{3}\right)_0^{30} = 27,000 - 9,000 = 18,000 \text{ .}$$