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Name:
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Instructor:
Section:

YEDİTEPE UNIVERSITY
MATH 172 FINAL EXAM

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: There are 5 questions on 4 pages. Solve all of them. Duration is 90 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1) **(a)** Find the critical point(s) of the function (10 points)

$$f(x,y) = x^2 + y^2 - \sqrt{e}\,yx$$

Solution:

$$f_x(x,y) = 2x - \sqrt{e}y = 0$$
$$f_y(x,y) = 2y - \sqrt{e}x = 0$$
$$x = 0 \; and \; y = 0$$

There is only one critical point which is (0,0).

(b) Classify each critical point as a relative maximum, a relative minimum, or neither (10 points).

Solution:

$$f_{xx}(x,y) = 2$$
$$f_{yy}(x,y) = 2$$
$$f_{xy} = -\sqrt{e}$$
$$D = f_{xx}f_{yy} - f_{xy}^2 = 4 - e > 0$$
$$f_{xx}(x,y) = 2 > 0$$

f(x,y) has a relative minimum (absolute minimum) at (0,0).

2) If $q_A = 1000 - e^{p_A} + p_B$ and $q_B = 400 + 2p_A - \sqrt[3]{12}\,p_B$, where q_A and q_B are the number of units demanded of products A and B, respectively, and p_A and p_B are their respective prices per unit, determine whether A and B are competitive products, complementary products, or neither (10 points).

<u>Solution</u>:

$$\frac{\partial q_A}{\partial p_B} = 1 > 0$$

$$\frac{\partial q_B}{\partial p_A} = 2 > 0$$

Therefore, A and B are competitive products.

3) Calculate the following indefinite integral (10 points)

$$\int \frac{x}{\sqrt{x+2}}dx = ?$$

<u>Solution</u>:

$$x+2=u \quad then \quad x=u-2$$

$$dx=du$$

$$\int \frac{x}{\sqrt{x+2}}dx = \int \frac{u-2}{\sqrt{u}}du = \int \frac{u}{\sqrt{u}}du - 2\int \frac{1}{\sqrt{u}}du$$

$$= \int (u)^{1/2}du - 2\int (u)^{-1/2}du = \frac{u^{3/2}}{3/2} - 2\frac{u^{1/2}}{1/2} + c$$

$$= \frac{2}{3}(x+2)^{3/2} - 4(x+2)^{1/2} + c$$

4) (a) Given $\ln(xy) = x^2y$, find $\frac{dy}{dx}$ (15 points).

Solution: Differentiate both sides with respect to x we get:

$$\frac{1}{xy}(y + x\frac{dy}{dx}) = 2xy + x^2\frac{dy}{dx}$$

$$\frac{1}{x} - 2xy = \frac{dy}{dx}(x^2 - \frac{1}{y})$$

$$\frac{dy}{dx} = \frac{\frac{1}{x} - 2xy}{x^2 - \frac{1}{y}}$$

(b) If $z = \sqrt{2x+3y}$ where $x = 3t + 2s$ and $y = t^2 + e^{2t} + s^2$ evaluate $\frac{\partial z}{\partial t}$ when $t = 0$ and $s = 1$ (15 points).

<u>Solution</u>:

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \left(\frac{1}{2}(2x+3y)^{-1/2}.2\right)(3) + \left(\frac{1}{2}(2x+3y)^{-1/2}.3\right)\left(2t+2e^{2t}\right)$$

$$= \frac{3}{\sqrt{2x+3y}} + \frac{3\left(t+e^{2t}\right)}{\sqrt{2x+3y}} = \frac{3\left(1+t+e^{2t}\right)}{\sqrt{2x+3y}}$$

$t = 0$ *and* $s = 1$ *then* $x = 2$ *and* $y = 2$

$$\frac{\partial z}{\partial t} = \frac{3\left(1+t+e^{2t}\right)}{\sqrt{2x+3y}} = \frac{3(1+0+1)}{\sqrt{2.2+3.2}} = \frac{6}{\sqrt{10}}$$

5) **(a)** By using matrix reduction, solve the given system and determine whether it has unique solution or infinitely many solutions (15 Points).

$$3x + 3y - z = 1$$
$$-2x - 2y + z = 1$$
$$-4x - 5y + 2z = 1$$

Solution:

$$\left(\begin{array}{ccc|c} 3 & 3 & -1 & 1 \\ -2 & -2 & 1 & 1 \\ -4 & -5 & 2 & 1 \end{array}\right) \xrightarrow[-2R_2+R_3\to R_3]{R_2+R_1\to R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & 1 \\ 0 & -1 & 0 & -1 \end{array}\right) \xrightarrow[R_3+R_1\to R_1]{2R_1+R_2\to R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ 0 & -1 & 0 & -1 \end{array}\right)$$

$$\xrightarrow[R_2\to R_3]{-R_3\to R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

The number of equation is the same as the unknowns; therefore it has unique solution as $x = 1, y = 1\ and\ z = 5$.

(b) Solve the given system below by using the inverse of its coefficient matrix (15 Points).

$$\begin{cases} 2x - 3y + 1 = 0 \\ y - x - 1 = 0 \end{cases}$$

Solution:

The equation can be written in matrix form as:

$$\mathbf{AX} = \mathbf{B} \quad \Rightarrow \quad \mathbf{A} = \begin{bmatrix} 2 & -3 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad and \quad \mathbf{B} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

The inverse of the coefficient matrix **A** can be found as follows:

$$\left[\mathbf{A}|\mathbf{I}\right] = \left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{R_1+R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 1 \end{array}\right]$$

$$\xrightarrow{-2R_2} \left[\begin{array}{cc|cc} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & -2 \end{array}\right] \xrightarrow{\frac{3}{2}R_2+R_1} \left[\begin{array}{cc|cc} 1 & 0 & -1 & -3 \\ 0 & 1 & -1 & -2 \end{array}\right] = \left[\mathbf{I}|\mathbf{A}^{-1}\right]$$

Hence we may solve the system of equations using the inverse of **A** matrix by

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

The solution set is {-2,-1}.