YEDİTEPE UNIVERSITY

MATH172 MIDTERM EXAM Summer 2013 **16.07.2013**

Name: **Student ID:**

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: There are 5 questions on 4 pages. Solve all of them. Duration is 60 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1) Evaluate the following integrals. (10 points each)

a) $\int \frac{(\sqrt{x}-1)^5}{\sqrt{x}}dx$

Solution:

$$\int \frac{(\sqrt{x}-1)^5}{\sqrt{x}}dx = 2\int u^5 du = 2\frac{u^6}{6} + C = \frac{(\sqrt{x}-1)^6}{3} + C \qquad \begin{cases} u = \sqrt{x}-1 \\ du = \frac{1}{2\sqrt{x}}dx \end{cases}$$

b) $\int \left(\frac{x-1}{x}\right)dx$

Solution:

$$\int \left(\frac{x-1}{x}\right)dx = \int \left(1-\frac{1}{x}\right)dx = x - \ln|x| + C$$

2) Find the following integrals (15 points each)

a) $\int \frac{x^2+1}{x^2(x-1)}dx$

Solution:

$$\frac{x^2+1}{x^2(x-1)} = \frac{A}{\underset{x(x-1)}{x}} + \frac{B}{\underset{x-1}{x^2}} + \frac{C}{\underset{x^2}{x-1}} = \frac{Ax(x-1)+B(x-1)+Cx^2}{x^2(x-1)}$$

$$x^2+1 = Ax(x-1)+B(x-1)+Cx^2$$

$$x=0 \quad \Rightarrow \quad 0^2+1 = A(0)(0-1)+B(0-1)+C(0^2) = -B \Rightarrow \quad B=-1$$

$$x=1 \quad \Rightarrow \quad 1+1 = A(1)\cancel{(1-1)}^0 + B\cancel{(1-1)}^0 + C(1^2) = C \Rightarrow \quad C=2$$

$$x=-1 \quad \Rightarrow \quad (-1)^2+1 = A(-1)(-1-1)+B(-1-1)+C(-1)^2 = 2A+2+2 \Rightarrow A=-1$$

$$\int \frac{x^2+1}{x^2(x-1)}dx = \int \frac{-1}{x}dx + \int \frac{-1}{x^2}dx + \int \frac{2}{x-1}dx$$

$$= -\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$$

b) $\int \frac{x}{e^{-x}}dx$

Solution:

$$\int \frac{x}{e^{-x}}dx = xe^x - \int e^x dx = xe^x - e^x + C \qquad \begin{cases} u = x & \Rightarrow \quad du = dx \\ dv = e^x dx & \Rightarrow \quad v = e^x \end{cases}$$

3. **Find the area of the region bounded by the parabola** $y = 2x^2 - 6$ **and the line** $y = -2 + 2x$ **(20 points).**

Solution

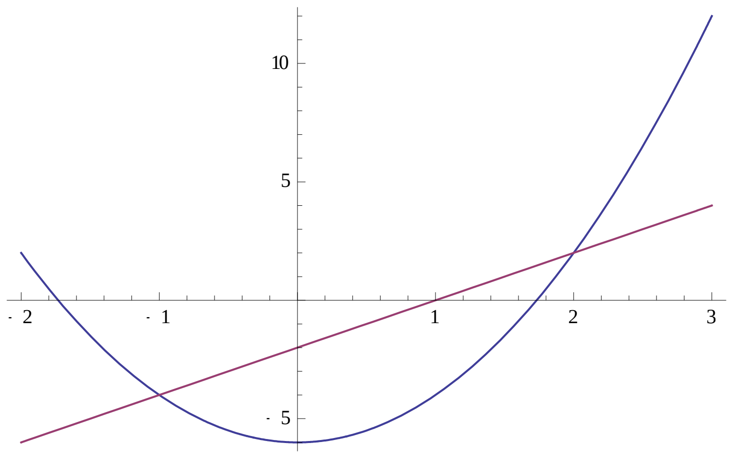
To find the intersections of two curves,

$$2x^2 - 6 = -2 + 2x \qquad \Rightarrow \qquad 2x^2 - 2x - 4 = 2(x+1)(x-2) = 0$$

$$x = -1 \qquad\qquad x = 2$$

$$\int_{-1}^{2} \left(-2 + 2x - 2x^2 + 6\right) dx = \int_{-1}^{2} \left(-2x^2 + 2x + 4\right) dx$$

$$= -\frac{2}{3}x^3 + \not{2}\frac{x^2}{\not{2}} + 4x \Big|_{-1}^{2} = -\frac{16}{3} + 4 + 8 - \left(\frac{2}{3} + 1 - 4\right) = 9$$



4. If the marginal reveue function for a manufacturer's product is $\frac{dr}{dq}=60-\frac{50q}{q+1}$. Find the demand function p (15 points).

Solution

$$\frac{dr}{dq}=60-\frac{50q}{q+1}$$

$$r(q)=\int\frac{dr}{dq}dq=\int\left(60-\frac{50q}{q+1}\right)dq=60\int dq-50\int\frac{q}{q+1}dq=60q-50\int\left(1-\frac{1}{q+1}\right)dq$$

$$=60q-50\left(q-\ln(q+1)\right)+C=10q+50\ln(q+1)+C$$

When no units are sold, there is no revenue then

$$r(0)=10(0)+50\ln(0+1)+C=0\Rightarrow C=0$$

$$r(q)=10q+50\ln(q+1)$$

To find the demand function,

$$p(q)=\frac{r(q)}{q}=\frac{10q+50\ln(q+1)}{q}=10+\frac{50\ln(q+1)}{q}$$

5. The demand equation for a product is $p\left(q+2\right)-2=0$ and the supply equation is $q-3p+1=0$

Determine the CONSUMERS' SURPLUS (*CS*) under the market equilibrium (15 points).

Solution:

From the demand equation we may pull the *p* price per unit as $p=\frac{2}{q+2}$ and from the supply equation $p=\frac{q+1}{3}$.

First we have to find the market equilibrium where the demand and supply functions are equal each other as follows:

$$f(q_0)=g(q_0)$$

$$\frac{2}{q_0+2}=\frac{q_0+1}{3}\qquad\Rightarrow\qquad q_0^2+3q_0-4=\left(q_0+4\right)\left(q_0-1\right)=0$$

$$\Rightarrow\qquad q_0=1\qquad\Rightarrow\qquad p_0=\frac{2}{q_0+2}=\frac{2}{3}$$

Then we can find the consumers' surplus (*CS*):

$$CS=\int_0^1\left(\frac{2}{q+2}-\frac{2}{3}\right)dq=\left(2\ln(q+2)-\frac{2}{3}q\right)_0^1=2\ln 3-\frac{2}{3}-2\ln 2=2\ln\frac{3}{2}-\frac{2}{3}$$