YEDİTEPE UNIVERSITY

MATH172 MIDTERM EXAM Summer 2014 **22.07.2014**

Name: **Student ID:**

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: There are 5 questions on 4 pages. Solve all of them. Duration is 60 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1) Evaluate the following integrals. (10 points each)

a) $\int \frac{(2-\sqrt{x})^9}{\sqrt{x}}dx$

Solution:

$$\int \frac{(2-\sqrt{x})^9}{\sqrt{x}}dx = -2\int u^9 du = -2\frac{u^{10}}{10} + C = -\frac{(2-\sqrt{x})^{10}}{5} + C \qquad \begin{cases} u = 2-\sqrt{x} \\ du = -\frac{1}{2\sqrt{x}}dx \end{cases}$$

b) $\int \left(\frac{x-2}{x-1}\right)dx = ?$

Solution:

$$\int \left(\frac{x-2}{x-1}\right)dx = \int \frac{u-1}{u}du = \int \left(1-\frac{1}{u}\right)du = x - \ln|x-1| + C \qquad \begin{cases} u = x-1 \\ du = dx \end{cases}$$

2) Find the following integrals (15 points each)

a) $\displaystyle\int \frac{x^2+1}{x(1-x)}dx$

Solution:

$$\begin{array}{r|l} & -x^2+x \\ x^2+1 & -1 \\ \hline \mp x^2 \pm x & \\ x+1 & \end{array}$$

$\Rightarrow \quad \dfrac{x^2+1}{x(1-x)} = \dfrac{x^2+1}{x-x^2} = -1 + \dfrac{x+1}{x-x^2}$

$$\frac{x+1}{x(1-x)} = \frac{A}{\underset{(1-x)}{x}} + \frac{B}{\underset{(x)}{1-x}} = \frac{A(1-x)+Bx}{x(1-x)}$$

$$x+1 = A(1-x)+Bx$$

$$x=0 \quad \Rightarrow \quad 0+1 = A(1-0)+B(0) = A \quad \Rightarrow \quad A=1$$

$$x=1 \quad \Rightarrow \quad 1+1 = A(1-1)+B(1) = B \quad \Rightarrow \quad B=2$$

$$\int \frac{x^2+1}{x(1-x)}dx = \int\left(-1+\frac{x+1}{x(1-x)}\right)dx = \int\left(-1+\frac{1}{x}+\frac{2}{1-x}\right)dx$$

$$= -x + \ln|x| - 2\ln|1-x| + C$$

b) $\displaystyle\int xe^{-x}dx$

Solution:

$$\int xe^{-x}dx = x(-e^{-x}) - \int -e^{-x}dx = -xe^{-x} - e^{-x} + C = -(x+1)e^{-x} + C$$

$$\begin{cases} u = x & \Rightarrow \quad du = dx \\ dv = e^{-x}dx & \Rightarrow \quad v = -e^{-x} \end{cases}$$

3. **Find the area of the region bounded by the parabola** $y + x^2 - 2 = 0$ **and the line** $y - x = 0$ **(20 points).**

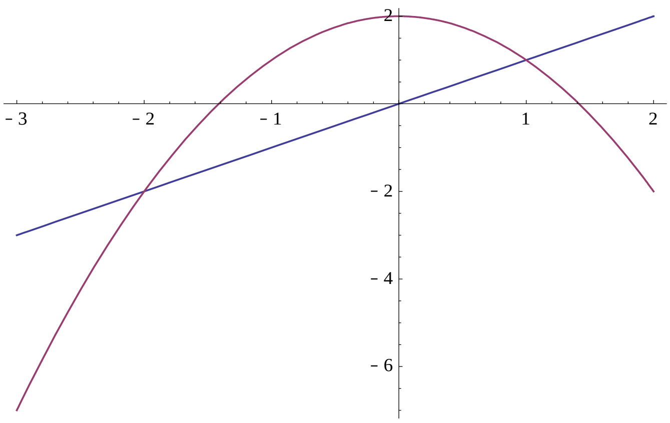
Solution

To find the intersections of two curves,

$$-x^2 + 2 = x \quad \Rightarrow \quad x^2 + x - 2 = (x-1)(x+2) = 0$$

$$x = 1 \qquad x = -2$$

$$\int_{-2}^{1} \left(-x^2 + 2 - x\right) dx = -\frac{x^3}{3} - \frac{x^2}{2} + 2x \Big|_{-2}^{1} = -\frac{1}{3} - \frac{1}{2} + 2 - \left(-\frac{-8}{3} - \frac{4}{2} - 4\right) = \frac{9}{2}$$



4. If the marginal revenue function for a manufacturer's product is $\frac{dr}{dq} = 60 - \frac{50q}{q+1}$. Find the demand function p (15 points).

Solution

$$\frac{dr}{dq} = 60 - \frac{50q}{q+1} = \frac{10q+60}{q+1} = 10\frac{q+6}{q+1}$$

$$r(q) = \int \frac{dr}{dq}dq = 10\int\left(\frac{q+6}{q+1}\right)dq = 10\int\left(1+\frac{5}{q+1}\right)dq = 10\int dq + 50\int \frac{1}{q+1}dq = 10q + 50\ln(q+1) + C$$

When no units are sold, there is no revenue then

$$r(0) = 10(0) + 50\ln(0+1) + C = 0 \Rightarrow C = 0$$

$$r(q) = 10q + 50\ln(q+1)$$

To find the demand function,

$$p = \frac{r(q)}{q} = \frac{10q + 50\ln(q+1)}{q} = 10 + \frac{50\ln(q+1)}{q}$$

5. The demand equation for a product is $p(q+2) - 2 = 0$ and the supply equation is $q - 3p + 1 = 0$

Determine the PRODUCERS' SURPLUS (*PS*) under the market equilibrium (15 points).

Solution:

From the demand equation we may pull the *p* price per unit as $p = \frac{2}{q+2}$ and from the supply equation $p = \frac{q+1}{3}$.

First we have to find the market equilibrium where the demand and supply functions are equal each other as follows:

$$f(q_0) = g(q_0)$$

$$\frac{2}{q_0+2} = \frac{q_0+1}{3} \qquad \Rightarrow \qquad q_0^2 + 3q_0 - 4 = (q_0+4)(q_0-1) = 0$$

$$\Rightarrow \qquad q_0 = 1 \qquad \Rightarrow \qquad p_0 = \frac{2}{q_0+2} = \frac{2}{3}$$

Then we can find the producers' surplus (*PS*):

$$PS = \int_0^1 \left(\frac{2}{3} - \frac{q+1}{3}\right)dq == \int_0^1 \left(\frac{1-q}{3}\right)dq = \frac{1}{3}\left(q - \frac{q^2}{2}\right)_0^1 = \frac{1}{6}$$