18.04.2013

Name:

Student ID:

Instructor:

Section:

YEDİTEPE UNIVERSITY

MATH 172 MIDTERM EXAM

Q1	Q2	Q3	Q4	Q5	Total

ATTENTION: There are 5 questions on 4 pages. Solve all of them. Duration is 60 minutes. Simply giving a final result is not sufficient to answer any question, so show all the steps you pursued to get any final result. Otherwise your answer will not be evaluated as a correct answer.

1) Evaluate the following integrals. (10 points each)

a) $\int x^2(x^3+4)^5 dx$

Solution: Let $u = x^3 + 4$. Then $du = 3x^2 dx$.

$$\text{So } \int x^2(x^3+4)^5 dx = \int \frac{1}{3}u^5 du = \frac{1}{18}u^6 + C = \frac{1}{18}(x^3+4)^6 + C$$

b) $\int \frac{x}{x+5} dx$

$$\text{Solution: } \int \frac{x}{x+5}dx = \int 1 - \frac{5}{x+5}dx = x - 5\ln|x+5| + C$$

2) Find the following integrals (10 points each)

a) $\int \frac{x+2}{(x-1).(x^2+2)} dx$

Solution:

$$\frac{x+2}{(x-1).(x^2+2)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2}$$

(x^2+2) $(x-1)$

$x+2 = A(x^2+2) + (Bx+C).(x-1)$

$x+2 = Ax^2 + 2A + Bx^2 - Bx + Cx - C$

$x+2 = (A+B)x^2 + (C-B)x + (2A-C)$

$A + B = 0,$ $C - B = 1$

$A = -B$ $2A - C = 2$

$2A - B = 3$

$2A + A = 3 \Rightarrow 3A = 3$

$A = 1$

$A = 1, \quad B = -1, \quad C = 0$

$$\int \frac{x+2}{(x-1).(x^2+2)} dx = \int \frac{1}{x-1} dx + \int \frac{-x+0}{x^2+2} dx$$

$$= \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{2x}{x^2+2} dx$$

$$= \ln|x-1| - \frac{1}{2} \ln|x^2+2| + C$$

$$= \ln\left|\frac{x-1}{\sqrt{x^2+2}}\right| + C$$

b) $\int \sqrt{x}\, lnx dx$

Solution:

$lnx = u$ and $\sqrt{x}dx$=dv

Then

$\frac{dx}{x} = du$ and $\frac{2}{3}x^{3/2} = v$

Therefore,

$$\int \sqrt{x}\, lnx dx = \frac{2}{3}x^{3/2} lnx - \frac{2}{3}\int x^{3/2}\frac{dx}{x}$$

$$=\frac{2}{3}x^{3/2} lnx - \frac{2}{3}\int x^{\frac{1}{2}}\, dx$$

$$=\frac{2}{3}x^{3/2} lnx - \frac{2}{3}\left(\frac{x^{3/2}}{3/2}\right)+c$$

$$= \frac{2}{3}x^{3/2}\left(lnx - \frac{2}{3}\right)+c$$

3) Given $y'' = 12x^2 + 6x + 2$, $y'(1) = 10$ and $y(0) = 1$, find y (10 points).

Solution: $y' = 4x^3 + 3x^2 + 2x + C_1$.

$y'(1) = 9 + C_1 = 10$, so $C_1 = 1$.

$y = x^4 + x^3 + x^2 + x + C_2$

$y(0) = C_2 = 1$, so $y = x^4 + x^3 + x^2 + x + 1$

4)

a) **Find the area of the region bounded by the parabola $y = -3x^2 + 12$ and the line $y = 12 - 6x$ (20 points).**

Solution

$$\int_0^2 [(-3x^2 + 12) - (12 - 6x)]dx = 4$$

b) **Find the average value of the function $y = -3x^2 + 6x$ between $x = 0$ and $x = 2$ (10 points).**

Solution

$$\frac{1}{2-0}\int_0^2 (-3x^2 + 6x)dx = 2$$

5)

a) If the marginal reveue function for a manufacturer's product is $\frac{dr}{dq} = 40 - 0.02q + \frac{500}{q+1}$**.**

Find the demand function p **(10 points).**

Solution

$$\frac{dr}{dq} = 40 - 0.02q + \frac{500}{q+1}$$

$$r(q) = \int \frac{dr}{dq}dq = \int\left(40 - 0.02q + \frac{500}{q+1}\right)dq = 40q - 0.02\frac{q^2}{2} + 500\ln(q+1) + C$$

When no units are sold, there is no revenue then

$$r(0) = 40.0 - 0.02\frac{0^2}{2} + 500\ln(0+1) + C = 0 \Rightarrow C = 0$$

$$r(q) = 40q - 0.01q^2 + 500\ln(q+1)$$

To find the demand function,

$$p(q) = \frac{r(q)}{q} = \frac{40q - 0.01q^2 + 500\ln(q+1)}{q} = 40 - 0.01q + 500\frac{\ln(q+1)}{q}$$

b) The demand equation for a product is $p(q+2) - 2 = 0$ **and the supply equation is** $q - 3p + 1 = 0$ **Determine the producers' surplus (*PS*) under the market equilibrium (10 points).**

Solution:

From the demand equation we may pull the *p* price per unit as $p = \frac{2}{q+2}$ and from the supply equation $p = \frac{q+1}{3}$.

First we have to find the market equilibrium where the demand and supply functions are equal each other as follows:

$$f(q_0) = g(q_0)$$

$$\frac{2}{q_0+2} = \frac{q_0+1}{3} \quad \Rightarrow \quad q_0^2 + 3q_0 - 4 = (q_0+4)(q_0-1) = 0$$

$$\Rightarrow \quad q_0 = 1 \quad \Rightarrow \quad p_0 = \frac{2}{q_0+2} = \frac{2}{3}$$

Then we can find the producers' surplus (*PS*):

$$PS = \int_0^1 \underbrace{\left(\frac{2}{3} - \frac{q+1}{3}\right)}_{\frac{1}{3}(-q+1)} dq = \frac{1}{3}\left(-\frac{q^2}{2} + q\right)_0^1 = \frac{1}{3}\left(-\frac{1}{2} + 1\right) = \frac{1}{6}$$