Example 1

An adsorption study is set up in laboratory by adding a known amount of activated carbon to six flasks which contain 200 mL of an industrial waste. An additional flask containing 200 mL of waste but no carbon is run as a blank. Plot the Langmuir isotherm and determine the values of the constants.

Flask No	Mass of C (mg)	Volume in Flask (mL)	Final COD (mg C /L)
1	804	200	4.7
2	668	200	7.0
3	512	200	9.31
4	393	200	16.6
5	313	200	32.5
6	238	200	62.8
7	0	200	250

Solution

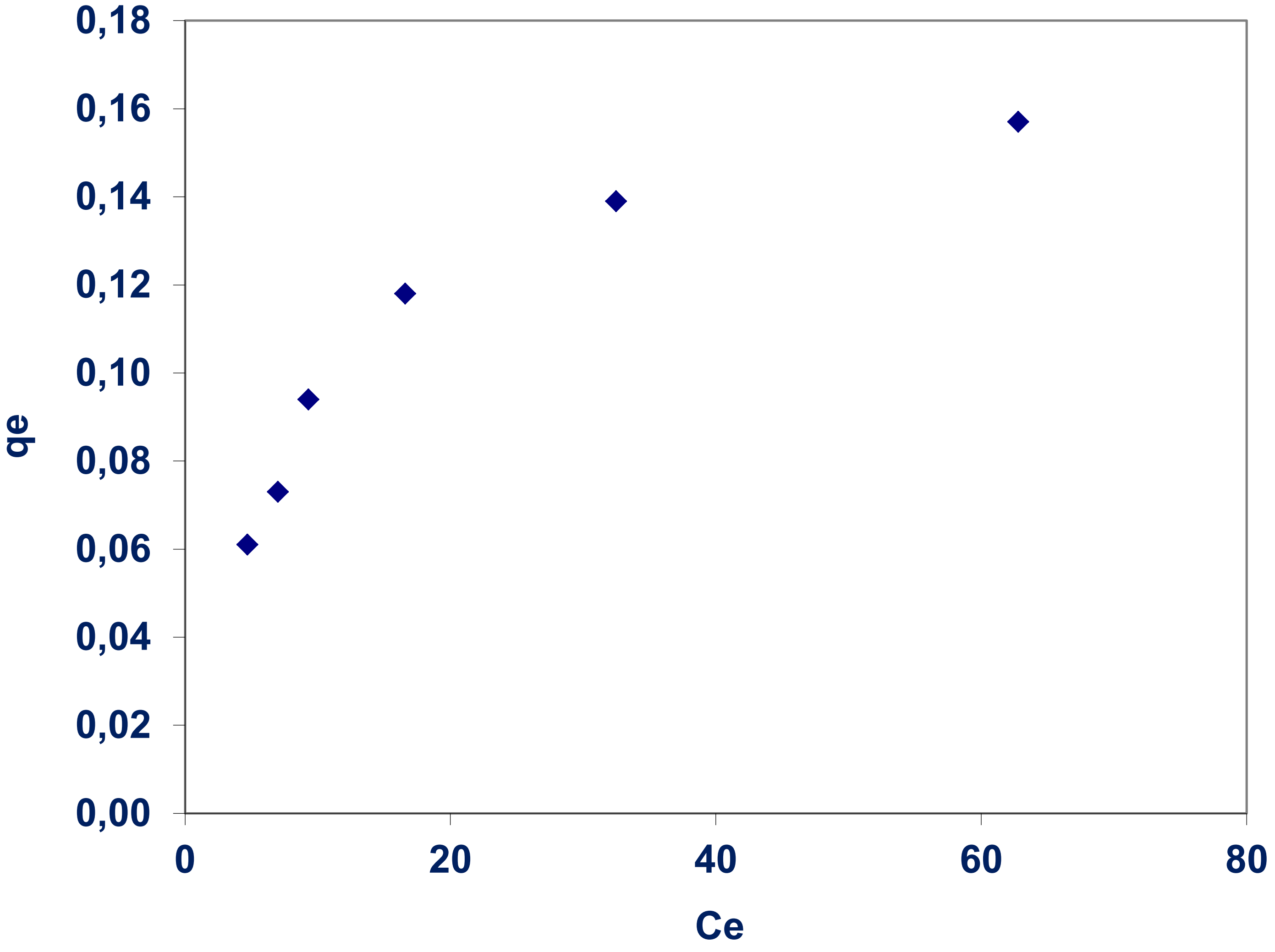
Flask 1

$$x=\frac{(250-4.7)\ \text{mg/L}.(200\ \text{mL})}{1000\ \text{mL/L}}=49.06\ \text{mg/L}$$

Amount of solute adsorbed

$$\frac{x}{m}=\frac{49.06\ \text{mg}}{804\ \text{mg}}=0.061\text{mg/mg}$$

Flask No	Mass of Adsorbate (mg)	x/m (mg/mg)
1	49.06	0.061
2	48.6	0.073
3	48.1	0.094
4	46.7	0.118
5	43.5	0.139
6	37.4	0.157
7	0	0


qe
0,18
0,16
0,14
0,12
0,10
0,08
0,06
0,04
0,02
0,00
0
20
40
60
80
Ce

Solution

Langmuir Isotherm

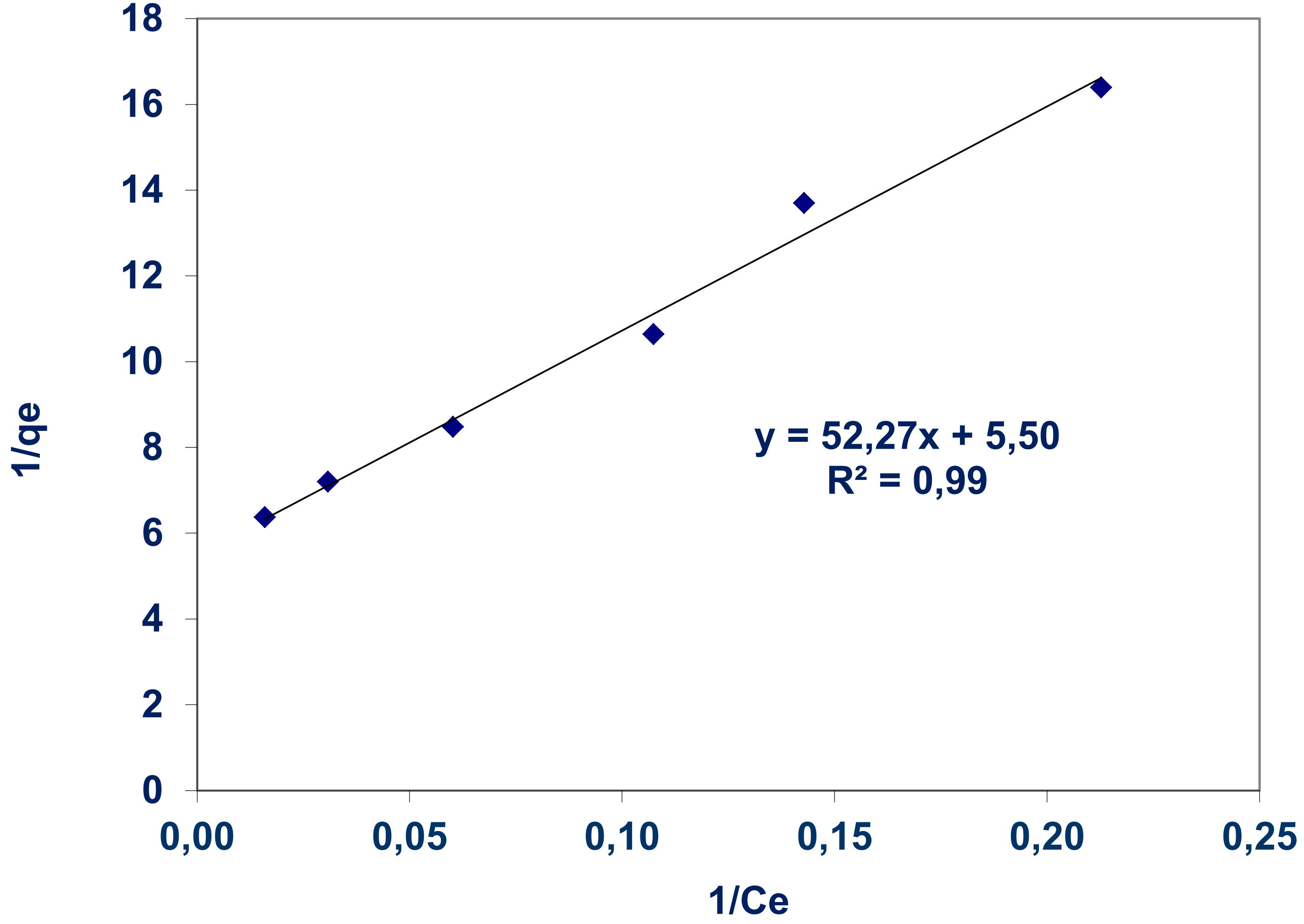
$$q_e = \frac{Q^0.K.C_e}{1+K.C_e}$$

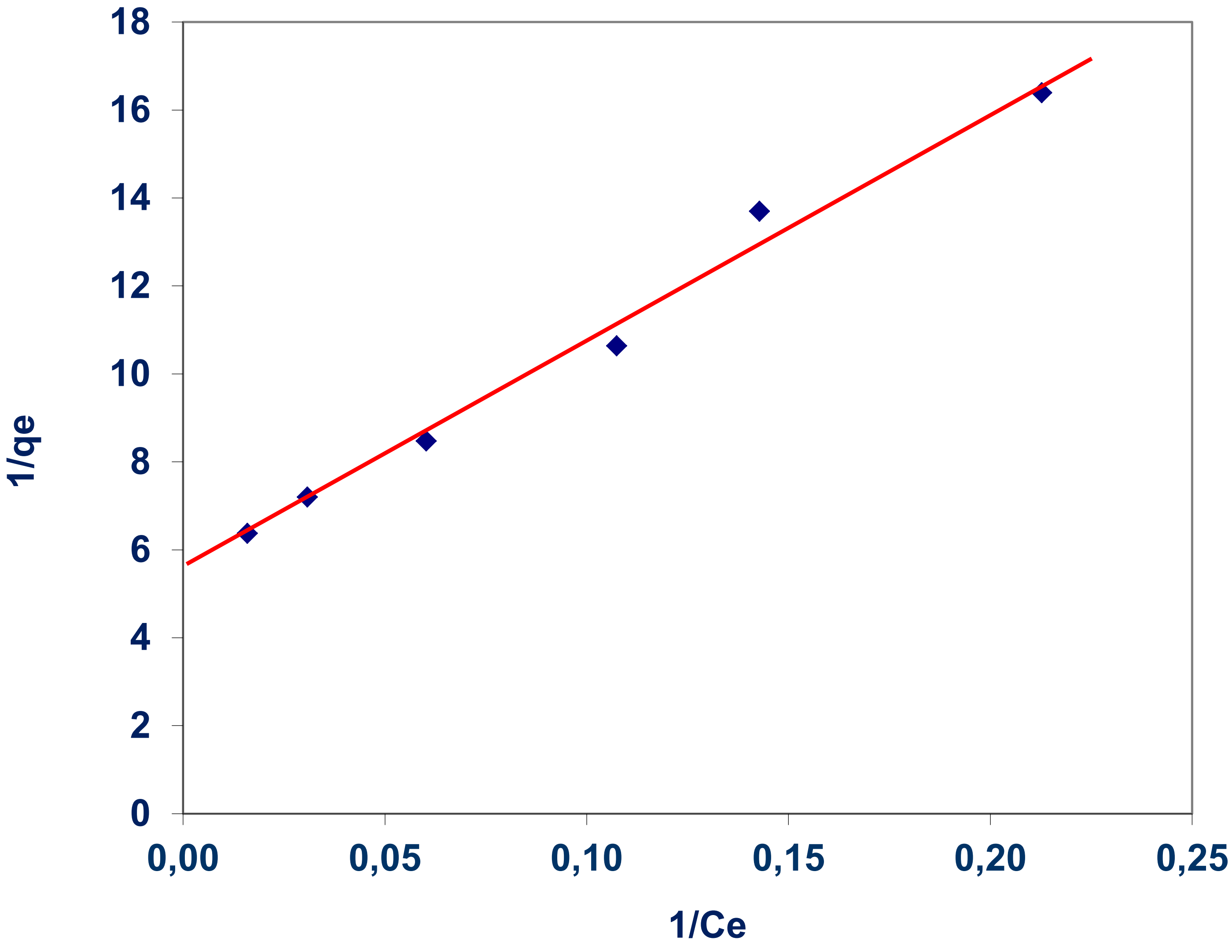
$$\frac{1}{q_e} = \frac{1}{Q^0} + \frac{1}{K \cdot Q^0} \cdot \frac{1}{C_e}$$

Determine the constants from a plot of 1/qe versus $1/C_e$

Flask No	$1/C_e$	$1/q_e$
1	0,21	16,39
2	0,14	13,70
3	0,06	10,64
4	0,03	8,47
5	0,02	7,19
6	0,02	6,37
7	0	0

Graphical Solution Using Excel





18
16
14
12
10
8
6
4
2
0
1/qe
0,00
0,05
0,10
0,15
0,20
0,25
1/Ce

Graphical Solution Without Computer

Read value of intercept = $\frac{1}{K} = 5.3$ $\quad K = \frac{1}{5.3} = 0.189$

Calculate slope = $\frac{16.39 - 7.19}{0.21 - 0.03} = 50.55$ $\quad \frac{1}{KQ^0} = 50.55$ $\quad Q^0 = 0.198$

The Langmuir Equation = $\frac{1}{q_e} = \frac{1}{0.189} + \frac{1}{(0.198)(0.189)C_e}$

Example 2

The following laboratory data were collected in a batch adsorption study. Plot the data according to Freundlich Isotherm and determine the values for the constants n and K_F. A volume of 500 mL is placed in each flask, and the waste has an initial COD of 100 mg/L.

Flask No	Mass of C (mg)	Volume in Flask (mL)	Final COD (mg C /L)
1	965	500	3.5
2	740	500	5.2
3	548	500	8.0
4	398	500	12.5
5	265	500	20.5
6	168	500	33
7	0	500	100

Solution

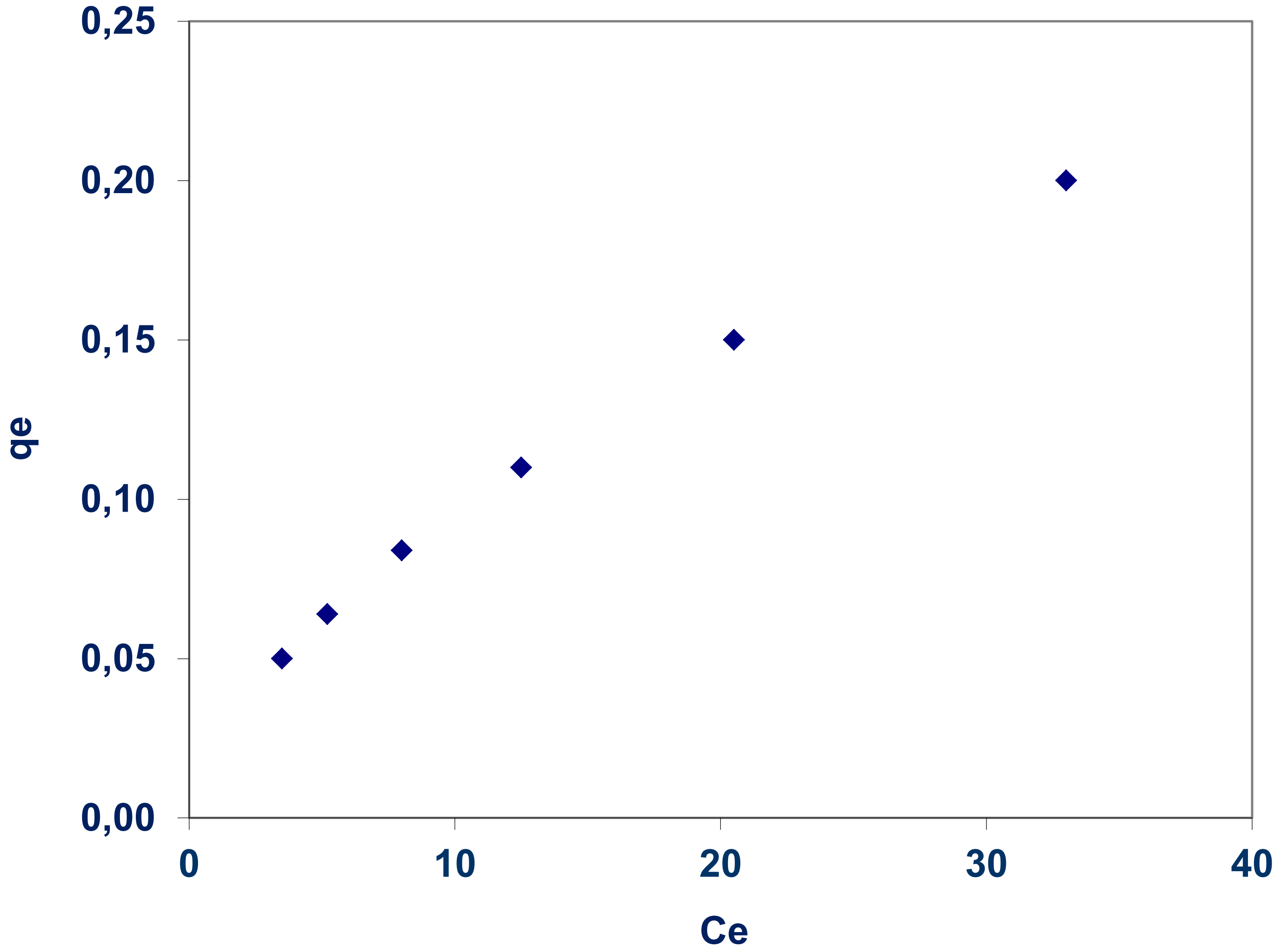
Flask 1

$$x = \frac{(100\text{-}3.5)\ \text{mg/L}\,(500\ \text{mL})}{1000\ \text{mL/L}} = 48.25\ \text{mg}$$

Amount of solute adsorbed

$$\frac{x}{m} = \frac{48.25\ \text{mg}}{965\ \text{mg}} = 0.05\,\text{mg/mg}$$

Flask No	Mass of Adsorbate (mg)	x/m (mg/mg)
1	48.25	0.05
2	47.40	0.064
3	46.0	0.084
4	43.75	0.11
5	39.75	0.15
6	33.5	0.20
7	0	0



0,25
0,20
0,15
0,10
0,05
0,00
qe
0
10
20
30
40
Ce

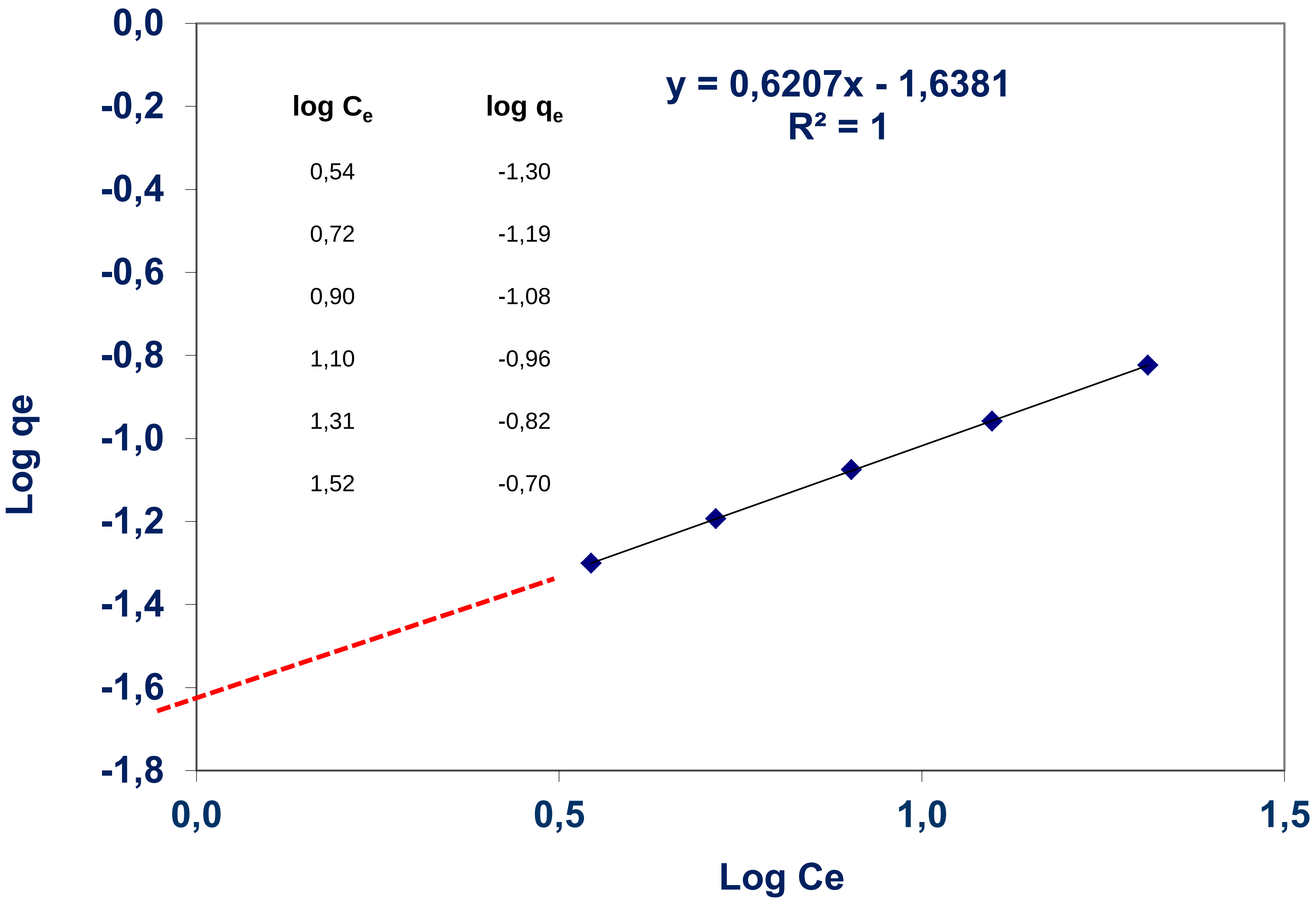
Solution

Freundlich Isotherm

$$q_e = K_F C_e^{1/n}$$

$$\log q_e = \log K_F + \frac{1}{n} \log C$$

Determine the constants from a plot of plot of Log q_e versus Log C_e.


0,0
-0,2
-0,4
-0,6
-0,8
-1,0
-1,2
-1,4
-1,6
-1,8
Log qe
0,0
0,5
1,0
1,5
Log Ce
y = 0,6207x - 1,6381
R² = 1
log Ce log qe
0,54 -1,30
0,72 -1,19
0,90 -1,08
1,10 -0,96
1,31 -0,82
1,52 -0,70

Calculate slope = $\frac{1}{n} = \frac{\log 0.064 - \log 0.11}{\log 12.5 - \log 5.20} = 0.618$

Slope = 0.62 = $\frac{1}{n}$

n = 1.61

Intercept = Log K_F = -1.64

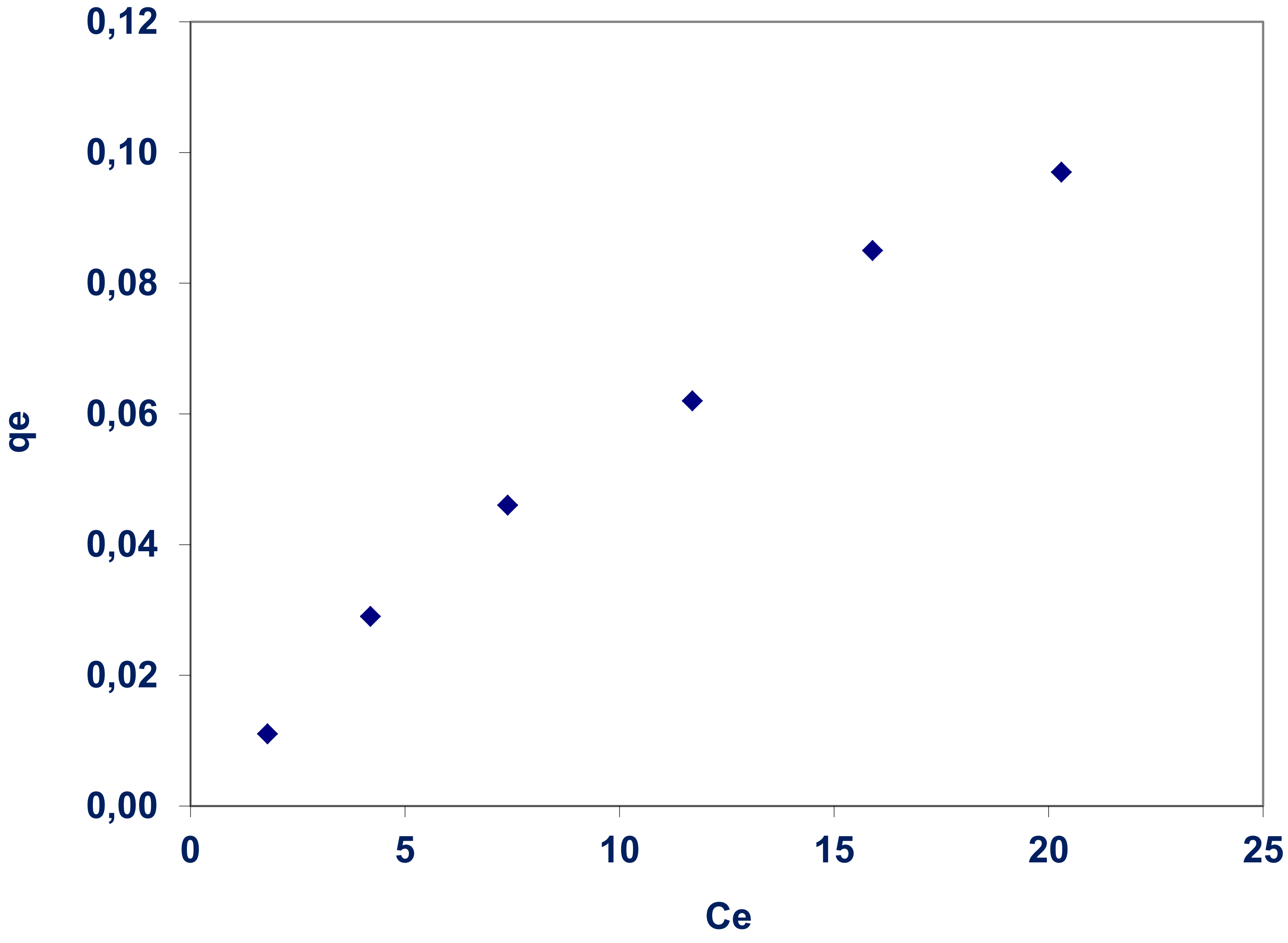
K_F = 0.023

$$q_e = K_F C_e^{1/n}$$

Example 3

Effluent water from a biological process is contacted with activated carbon and allowed to come to equilibrium. The equilibrium data in terms of total organic carbon (TOC) are given below. Determine the best values of the constants for the Langmuir and Freundlich isotherms.

C_e TOC in solution, mg/L	q_e TOC on Carbon, mg/mg C
1.8	0.011
4.2	0.029
7.4	0.046
11.7	0.062
15.9	0.085
20.3	0.097



0,12
0,10
0,08
0,06
0,04
0,02
0,00
qe
0
5
10
15
20
25
Ce

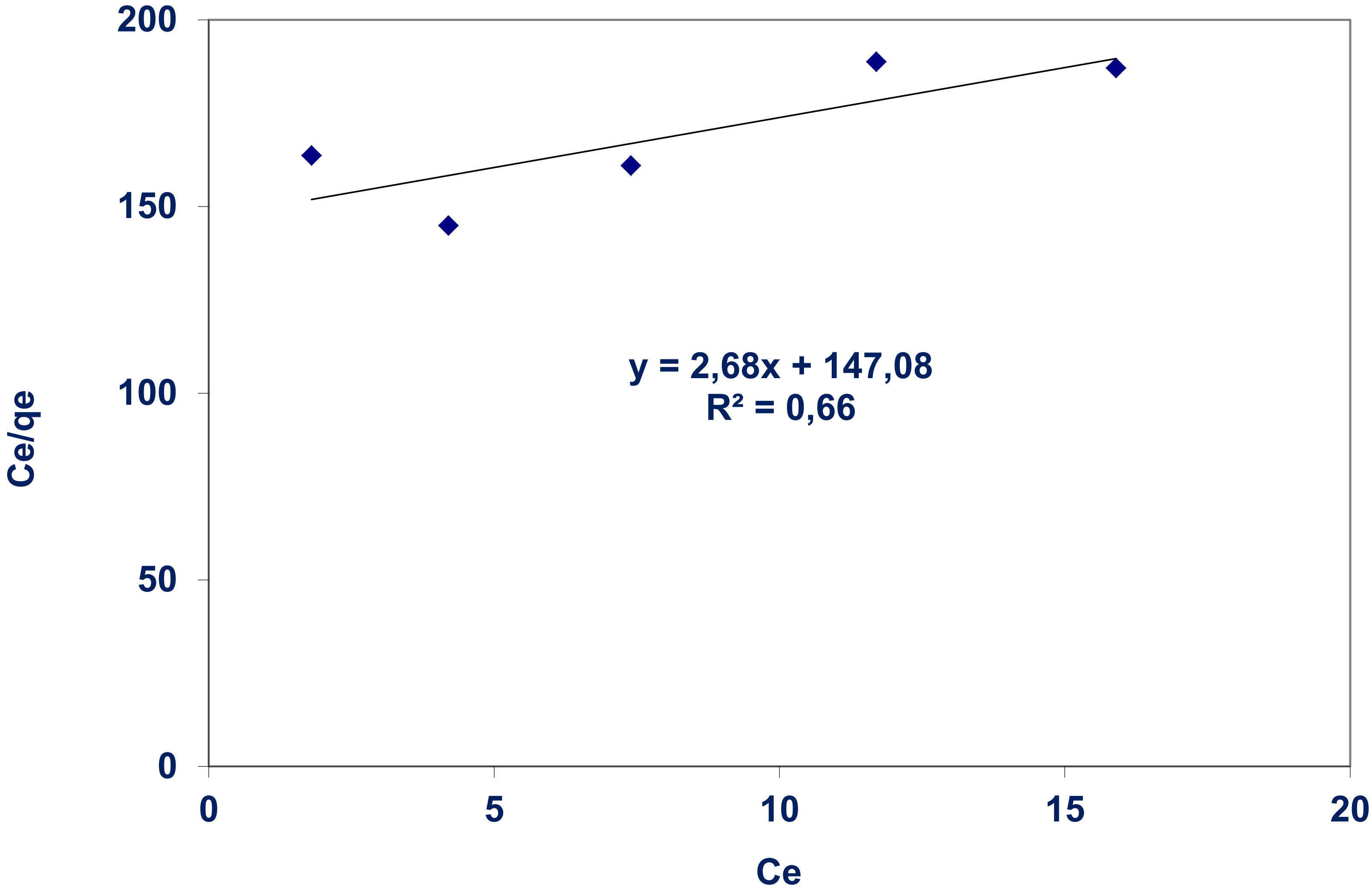
Solution

Langmuir Isotherm

$$q_e = \frac{Q^o . b . C_e}{1 + b . C_e}$$

$$\frac{C_e}{q_e} = \frac{1}{b . Q^o} + \frac{C_e}{Q^o}$$

Determine the constants from a plot of plot of C_e/q_e versus C_e.


200
150
100
50
0
Ce/qe
0
5
10
15
20
Ce
y = 2,68x + 147,08
R² = 0,66

Slope = $\frac{1}{Q^o} = \mathbf{2.7}$

Intercept = $\frac{1}{b.Q^o} = \mathbf{147}$

$Q^o = \mathbf{0.37}$ mg TOC / mg C

b = **0.018** L / mg TOC

$$q_e = \frac{Q^o.b.C_e}{1+b.C_e}$$

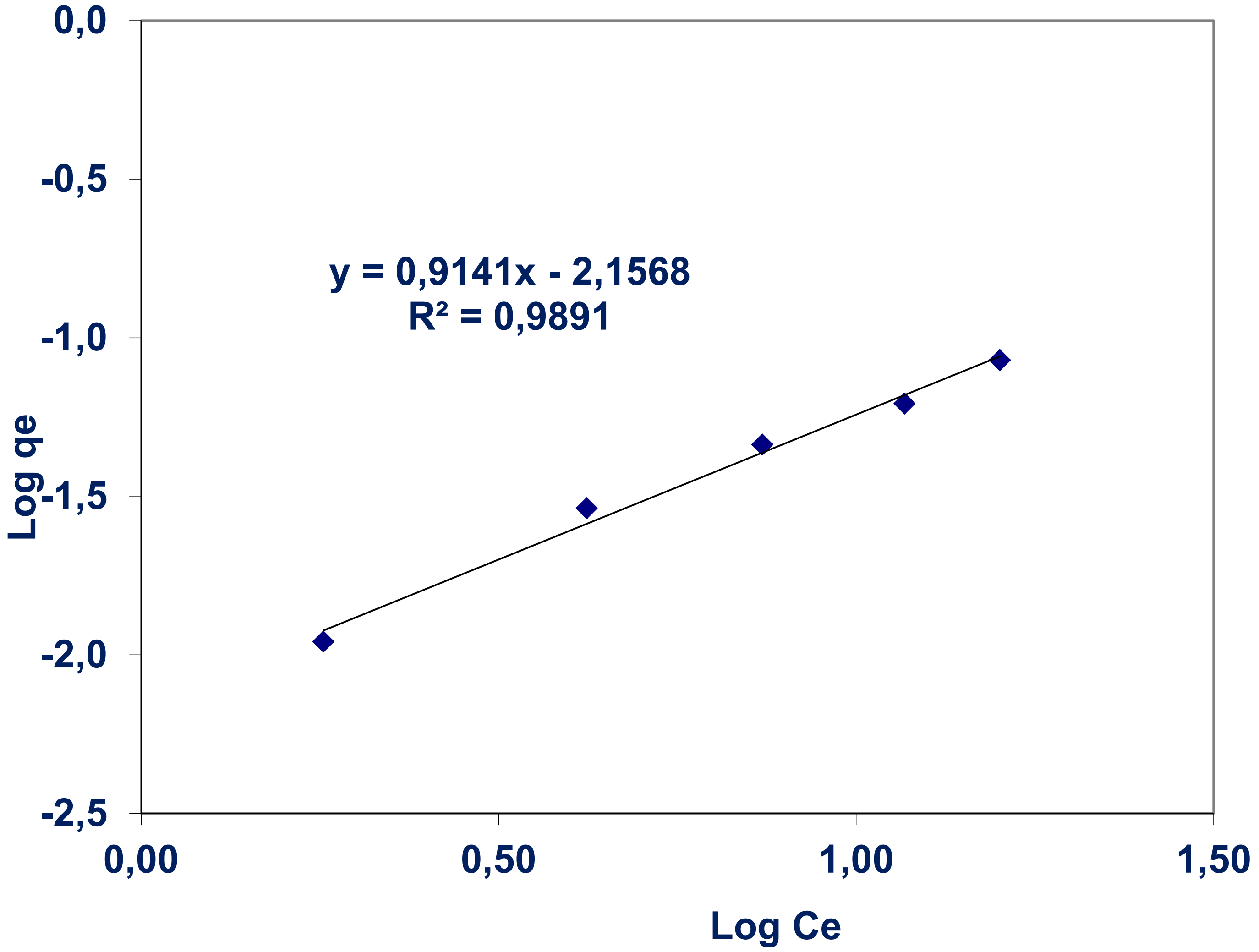
Solution

Freundlich Isotherm

$$q_e = K_F C_e^{1/n}$$

$$\log q_e = \log K_F + \frac{1}{n} \log C$$

Determine the constants from a plot of plot of Log q_e versus Log C_e.



0,0
-0,5
-1,0
-1,5
-2,0
-2,5
Log qe
y = 0,9141x - 2,1568
R² = 0,9891
0,00
0,50
1,00
1,50
Log Ce

Slope = 0.91 = 1/n

n = 1.1

Intercept = Log K_F = -2.16

K_F = 6.92 x 10^{-3}

$$q_e = K_F C_e^{1/n}$$

Example 4

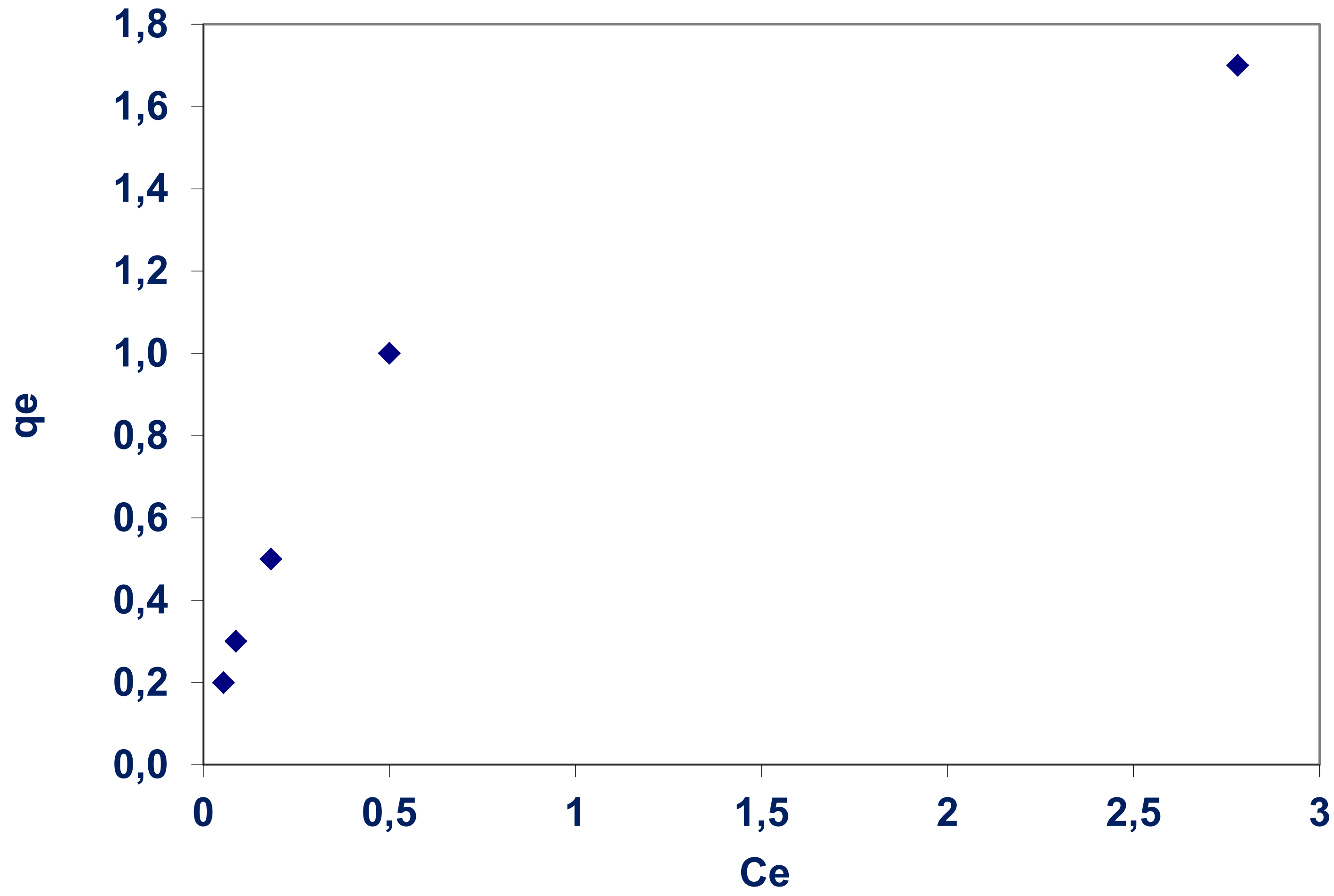
Iron – coated clays ("red clays") can adsorb phosphorus. A municipality is considering the use of these materials for the removal of P from the town's wastewater prior to discharge into a lake. The following adsorption data are obtained in the laboratory using the clay and secondary effluent from the town's treatment facility.

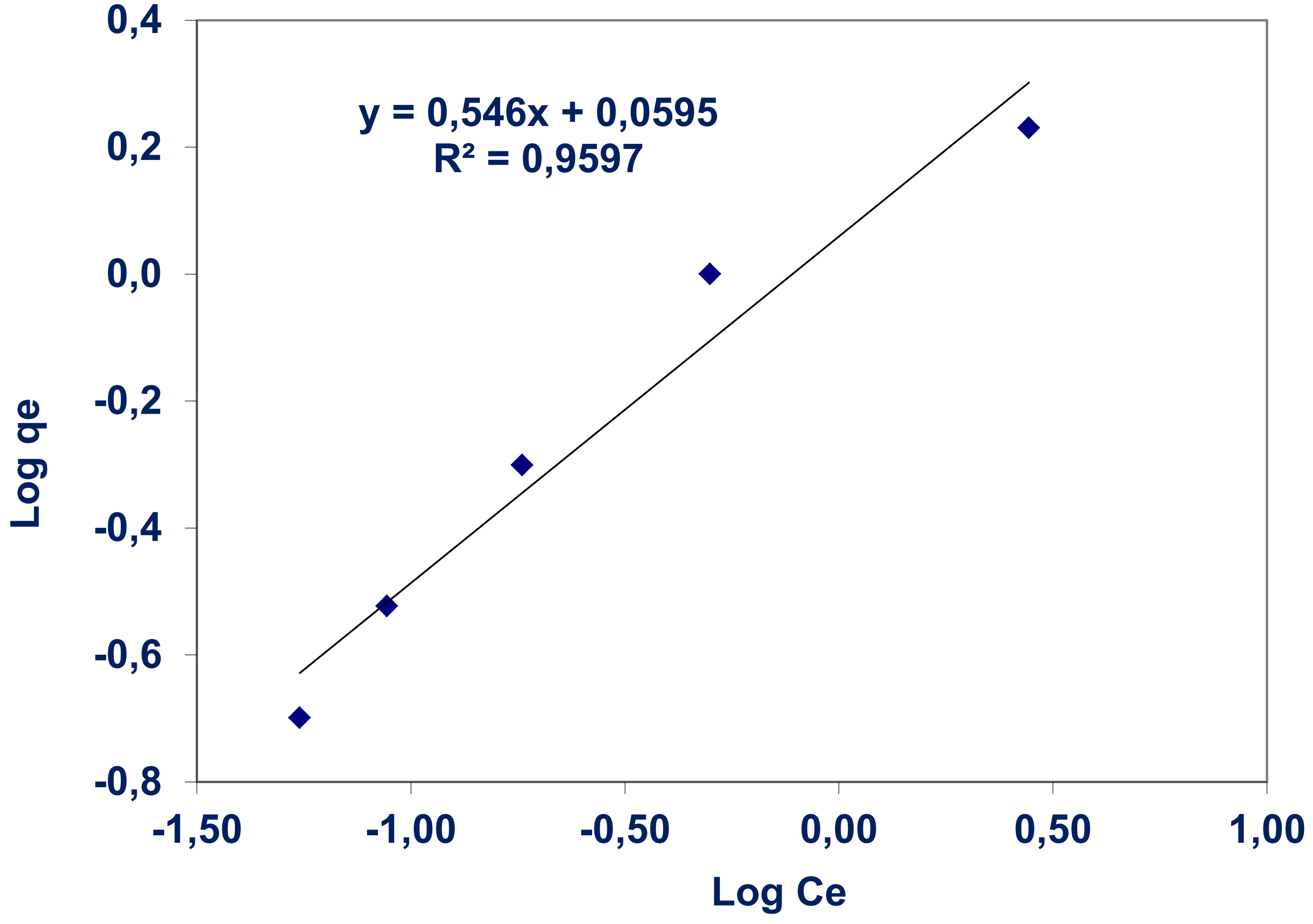
q_e P adsorbed, mg P / g clay	**C_e** Equilibrium P in solution, mg P / L
0.2	0.055
0.3	0.088
0.5	0.182
1.0	0.50
1.7	2.78

Example 4 cont'd

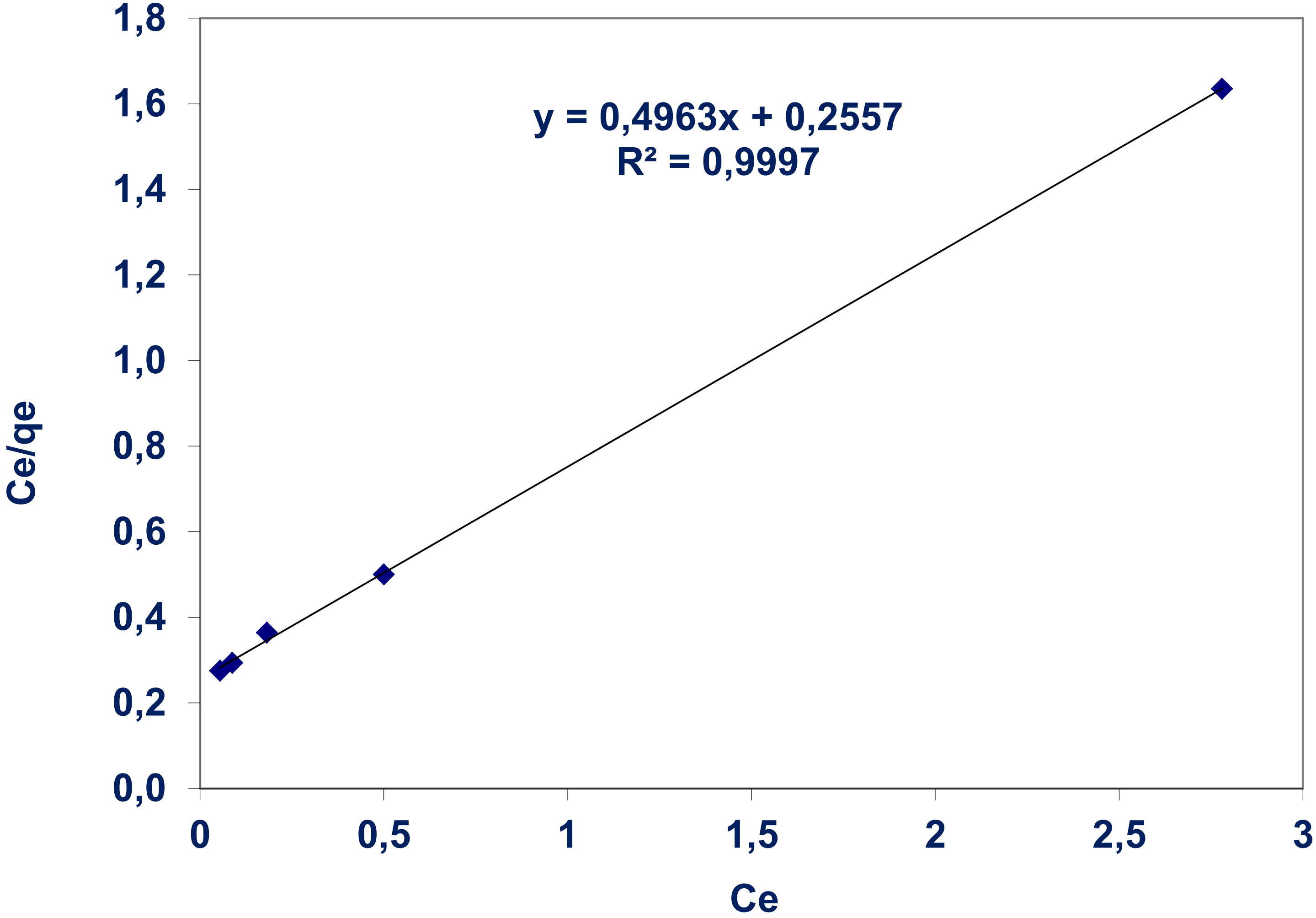
a) Determine the maximum P adsorption capacity of this clay (mg P / g clay).

b) The P concentration in the secondary effluent is 8 mg P/L and the flow is 3 MGD. Make a preliminary estimate of the clay required (tons/month) to remove 99% of P in a batch reactor. State any simplifying assumptions that you make. (MGD : million galloon/day)


1,8
1,6
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
qe
0
0,5
1
1,5
2
2,5
3
Ce


y = 0,546x + 0,0595
R² = 0,9597
0,4
0,2
0,0
-0,2
-0,4
-0,6
-0,8
Log qe
-1,50
-1,00
-0,50
0,00
0,50
1,00
Log Ce

C_e/q_e
0.275
0.293
0.364
0.5
1.635



Solution

a)

Slope of (C_e/q_e) versus C_e curve = $\frac{1}{Q^o} = 0.496$

$Q^o = \mathbf{2.02}$ **mg P / g clay**

Intercept of (C_e/q_e) versus Ce curve = $\frac{1}{b.Q^o} = 0.256$

$b.Q^o = 3.906 \quad b = \frac{3.906}{2.02} = \mathbf{1.94}$ **L / mg P**

Langmuir Isotherm

$$q_e = \frac{Q^0 . b . C_e}{1 + b . C_e}$$

$$q_e = \frac{2.02 \times 1.94 \times C_e}{1 + 1.94 \times C_e}$$

b) Clay required (tons/month) to remove 99% of P in a batch reactor.

What is C_e to be used in the above equation?

b) 99% P removal is req'd. Therefore, C_e=0.08 mg/L

$$q_e = \frac{(2.02)\,x\,(1.94)\,x\,(0.08)}{1+(1.94)\,x\,(0.08)} = \frac{0.314}{1.155} = \mathbf{0.272}\ \textbf{mg P / mg clay}$$

Total P removed in 1 day =

$$(8\text{mg/L-}0.08\text{mg/L})\,x3\text{MGD}x\,\frac{3.78x10^6\text{L}}{1\text{MG}} = \mathbf{89.8x10^6\,mg/d}$$

Total P removed in 1 month =

$$89.8x10^6\text{mg/d} \times 30\text{d} = \mathbf{2.69x10^9\,mg\,/\,month}$$

Total clay required (tons/month) =

$$\frac{2.69x10^9\text{mg P/month}}{0.272\text{mg P/gclay}}\,x\,\frac{1\text{ ton}}{10^6\text{ g}} = \mathbf{9900\ tons/month}$$

Assumptions made :

We assumed that this is a monolayer coverage equilibrium model, and all adsorption sites are equally probable.

However, in the real system there are going to be several different kinds of pollutants in the wastewater that will compete for adsorption site.

We also assumed that temperature and pH are the same in the real system as they are in the laboratory study.