ION EXCHANGE
DESIGN PROCEDURES
Design Procedures

• The breakthrough curves for an ion exchange column and an adsorption column are similar.
• The contacting techniques are almost identical.
• Therefore, the same procedures used for the design of adsorption columns may be used for ion exchange columns.
  o the scale-up approach
  o the kinetic approach
A laboratory- or pilot-scale break-through curve is required for both procedures. The breakthrough curve for a column shows the solute or ion concentration in the effluent on the y-axis versus the effluent throughput volume on the x-axis. The area above the breakthrough curve represents the amount of solute or ions taken up by the column and is

\[
\int (C_0 - C) dV
\]

from \( V = 0 \) to \( V = \) the allowable throughput volume under consideration. At the allowable breakthrough volume, \( V_B \), the area above the breakthrough curve is equal to the amount of ions removed by the column.
At complete exhaustion, \( C = C_o \) and the area above the breakthrough curve is equal to the maximum amount of ions removed by the column. At complete exhaustion, the entire exchange column is in equilibrium with the influent and effluent flows. Also, the ion concentration in the influent is equal to the ion concentration in the effluent.
Another design approach is to determine the meqs or equivalent weights of the ions removed by a test column using the breakthrough curve. The throughput volume under consideration should be the allowable break-through volume, $V_B$, at the allowable breakthrough concentration, $C_a$. The ratio of the amount of ions removed per unit mass of exchanger is computed.
Then, using this ratio, the mass of exchanger required is calculated from the allowable breakthrough volume for the design column and the concentration of the polyvalent metallic ions to be removed from the liquid flow. For this method to be valid, the flowrate used for the test column in terms of bed volumes per hour must be similar to the flowrate of the design column.
\[
\frac{C}{C_0} = \frac{1}{1 + e^\frac{k_1 Q (q_o M - C_0 V)}{Q}}
\]

- \(C\) = effluent solute concentration
- \(C_0\) = influent solute concentration
- \(k_1\) = rate constant
- \(q_o\) = maximum solid phase concentration of the sorbed solute \(\rightarrow\) e.g. mg/g
- \(M\) = mass of the adsorbent
- \(V\) = volume treated
- \(Q\) = flow rate
\[
1 + e^{\frac{k_t}{Q}(q_o M - C_o V)} = \frac{C_o}{C}
\]

taking natural log. of both sides yield the design equation,

\[
\ln\left(\frac{C_o}{C} - 1\right) = \frac{k_1 q_o M}{Q} - \frac{k_1 C_o V}{Q}
\]

\[
y = b + mx
\]

\[
y = \ln\left(\frac{C_o}{C} - 1\right), \quad x = V, \quad m = -\frac{k_1 C_o}{Q}, \quad b = \frac{k_1 q_o M}{Q}
\]
The breakthrough curves for an ion exchange column and an adsorption column are similar. A laboratory or pilot scale breakthrough curve is required.

A breakthrough curve shows the solute or ion concentration in the effluent on the y-axis versus the effluent throughout volume on the x-axis. The area above the breakthrough curve represents the amount of solute or ions taken up by the columns.

\[ \int (C_0 - C) \, dV \]

from \( V = 0 \) to \( V = \text{the allowable throughput volume} \).
Example 13.1 SI Ion Exchange in Waste Treatment

An industrial wastewater with 107 mg/L of Cu\(^{2+}\) (3.37 meq/L) is to be treated by an exchange column. The allowable effluent concentration, \(C_a\), is 5% \(C_o\). A breakthrough curve, shown in Figure 13.6, has been obtained from an experimental laboratory column on the sodium cycle. Data concerning the column are as follows: inside diameter = 1.3 cm, length = 45.7 cm, mass of resin = 41.50 g on a moist basis (23.24 gm on a dry basis), moisture = 44%, bulk density of resin = 716.5 kg/m\(^3\) on a moist basis, and liquid flowrate = 1.0428 L/d. The design column flowrate will be 378,500 L/d, the allowable breakthrough time is 7 days of flow, and the resin depth is approximately twice the column diameter. Using the kinetic approach to column design, determine:

1. The kilograms of resin required.
2. The diameter and depth.
3. The height of the sorption zone.
Solution

Co = 107 mg/L (3.37 meq/L)
Ca = 5% Co
d = 1.3 cm
L = 45.7 cm

\[ \text{Mass}_{\text{resin}} = 41.50 \text{ g on a moist basis} \]
\[ = 23.24 \text{ g on a dry basis} \]

moisture = 44%

\[ \text{bulk density}_{\text{resin}} = 716.5 \text{ kg/m}^3 \text{ on a moist basis} \]

\[ Q = 1.0428 \text{ L/d} \]
\[ Q_{\text{design}} = 378,500 \text{ L/d} \]

breakthrough time = 7 days

resin depth = 2 \times \text{ column diameter}
1. Mass of resin required

\[
\ln\left(\frac{C_o}{C} - 1\right) = \frac{k_1 q_o M}{Q} - \frac{k_1 C_o V}{Q}
\]
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\[ y = -0.7603x + 15.341 \]
Solution

Co = 107 mg/L (3.37 meq/L)

Ca = 5% Co

d = 1.3 cm

L = 45.7 cm

Mass_{resin} = 41.50 g on a moist basis

= 23.24 g on a dry basis

moisture = 44%

bulk density_{resin} = 716.5 kg/m^3 on a moist basis

Q = 1.0428 L/d

Q_{design} = 387,500 L/d

breakthrough time = 7 days

resin depth = 2 x column diameter
\[ 0.7603 \, \text{L}^{-1} = \frac{k_1 C_o}{Q} \]

\[ k_1 = \left( 0.7603 \, \text{L}^{-1} \right) \times \left( 1.0428 \, \frac{\text{L}}{d} \right) \times \left( \frac{1 \, \text{L}}{3.37 \, \text{meq}} \right) \]

\[ k_1 = 236 \, \text{L/d.eq} \]

\[ 15.341 = \frac{k_1 q_o M}{Q} \quad \iff \quad 15.341 = \frac{(236 \, \text{L/d.eq}) \times q_o \times 23.24 \, \text{g}}{1.0428 \, \text{L/d}} \]
\[ q_o = \frac{(1.0428 \text{ L/d}) \times 15.341}{(236 \text{ L/d.eq}) \times (23.24 \text{ g})} = 2.92 \times 10^{-3} \frac{\text{eq}}{\text{g}} = 2.92 \frac{\text{eq}}{\text{kg}} \]

Compute the mass of resin required for the design column from :

\[ \ln \left( \frac{C_o}{C} - 1 \right) = \frac{k_1 q_o M}{Q} - \frac{k_1 C_o V}{Q} \]

\[ \ln \left( \frac{C_0}{0.05 C_o} - 1 \right) = \left( \frac{236 \text{ L}}{\text{eq.d}} \right) \times \left( \frac{2.92 \text{ eq}}{\text{kg}} \right) \times M_{\text{resin}} \times \frac{1 \text{ d}}{378500 \text{ L} - \frac{1 \text{ d}}{378500 \text{ L}} \times \left( \frac{236 \text{ L}}{\text{eq.d}} \right) \times \left( 3.37 \times 10^{-3} \text{ eq} \right) \times \left( \frac{378500 \text{ L}}{\text{d} \times 7 \text{ d}} \right) \]
\[ 2.94 = 1.82 \times 10^{-3} \, M - 5.57 \]

\[ M = 4676 \, \text{kg} \]

2. Diameter and depth:

\[ \text{Resin Volume} = (4676 \, \text{kg}) \times \frac{(41.5 \, \text{q wet wt.})}{(23.24 \, \text{q dry wt.})} \times \frac{1 \, \text{m}^3}{716.5 \, \text{kg}} = 11.65 \, \text{m}^3 \]

\[ \text{or} \]

\[ \frac{1 \, \text{g wet wt.}}{0.56 \, \text{g dry wt.}} \quad \text{because 44\% moisture} \]
Volume = \left( \frac{\pi D^2}{4} \right) \times (2D) = 11.65 \text{m}^3 \quad \Rightarrow \quad D = 1.95 \text{ m}

Depth = 2 \times 1.95 = 3.90 \text{ m}
3. The height of the sorption zone
   the length of the column in which adsorption occurs

Sorption zone, $Z_s$, is related to the

- column height, $Z$
- breakthrough volume, $V_B$
- volume of exhaustion, $V_T$
\[ Z_s = Z \left[ \frac{V_z}{V_T - 0.5V_z} \right] \text{ where } V_z = V_T - V_B \]

The exhaustion is considered to occur at \( C = 0.95 \, C_0 \)

\[ V_B = 378,500 \, \frac{L}{d} \times 7 \, d = 2.65 \times 10^6 \, L \]

\[
\ln \left( \frac{C_0}{0.95C_0} - 1 \right) = \left( \frac{236L}{\text{eq. d}} \right) \times \left( \frac{2.92 \, \text{eq}}{\text{kg}} \right) \times 4676 \, \text{kg} \times \frac{1 \, \text{d}}{378500 \, L} - \left( \frac{236L}{\text{eq. d}} \right) \\
\times \left( \frac{3.37 \times 10^{-3} \, \text{eq}}{L} \right) \times V_T \times \left( 378500 \, \frac{L}{d} \right)
\]
\[-2.94 = 8.51 - 2.1 \times 10^{-6} \ V_T\]

\[2.1 \times 10^{-6} \ V_T = 11.45\]

\[V_T = 5.45 \times 10^6 \ L\]

\[V_Z = \left(5.45 \times 10^6 \ L\right) - \left(2.65 \times 10^6 \ L\right) = 2.8 \times 10^6 \ L\]

\[Z_S = 4.04 \ m \times \left[\frac{2.8 \times 10^6}{5.45 \times 10^6 - 0.5 \times 2.8 \times 10^6}\right] = 2.79 \ m\]
Example
A home water softener has 0.1 m³ of ion – exchange resin with an exchange capacity of 57 kg/m³ (i.e., 57 kg of hardness as CaCO₃ per m³ of resin volume). The occupants use 2000 L of water per day. The water contains 280 mg/L of hardness as CaCO₃ and it is desired to soften it to 85 mg/L as CaCO₃. Assumption: All (100%) hardness in the water which passes through the ion exchange column is removed.

1. How much water should be bypassed?
2. What is the time between regeneration cycle “Breakthrough time”? 
Solution: 1. C: concentration
Q: flowrate
Loading rate = C.Q

Mass balance equations
Accumulation = Input - Output ± Reactions

*with no accumulation and no reaction*
Input = Outputs.

\[
(Q - Q_b) \cdot C_e + Q_b \cdot C_{in} = Q \cdot C_p
\]
If $C_e = 0$

$$Q_b \cdot C_{in} = Q \cdot C_p$$

$$\frac{Q_b}{Q} = \frac{C_p}{C_{in}} = \frac{85 \text{ mg/L}}{280 \text{ mg/L}} = 0.3 \quad 30\% \text{ of total flow}$$

$$Q_b = 0.3Q = 0.3 \times 2000 \text{ L/d} = 600 \text{ L/d}$$

$Loading rate = 0.7 \times Q \times C_{in} =$

$$= (0.7) \times (2000 \text{ L/d}) \times (280 \text{ mg/L}) = 392000 \text{ mg/d}$$
2. Breakthrough time =

\[
\frac{\text{Total capacity of resin}}{\text{Mass of ions removed/time}} = \frac{(57 \text{ kg/m}^3) \times (0.1 \text{ m}^3)}{(392000 \text{ mg/d}) \times (10^{-6} \text{ kg/mg})} = 14.5 \text{ d}
\]
Example:

An ion exchange process is to be used to soften water at the rate of 500 gpm. A synthetic zeolite resin will be packed in shells with diameter of 5 ft. The resin has an exchange capacity of 20 kilograins of CaCO₃ per ft³ when regenerated at the rate of 15 lb of salt per ft³. The raw water has total hardness of 100 mg/L as CaCO₃. Assume this process can achieve 95% hardness removal efficiency. (1 grain/gal = 17.1 mg/L). Use the design criteria

a) The maximum loading rate = 5 gpm/ft³ of resin.
b) The bed depth = 30-72 inches.
1. Calculate total hardness (mg/L as CaCO$_3$).
2. Calculate hardness to be removed (grains/gal).
3. Calculate hardness to be removed (kilograins/d).
4. Calculate total resin required (ft$^3$).
5. Calculate the amount of salt required for regeneration.
6. Calculate the bed depth and shell diameter for the ion exchange equipment (considering two units).