1- \( T(n) = T(n/3) + T(2n/3) + cn \), deterministic selection algorithm using median.

Solve recurrence for the running time of the algorithm by evaluating the depth of the recurrence tree.

Answer: The height of the tree is at least \( \log_3 n \), and at most \( \log_{3/2} n \) and the sum of the costs in each level is \( n \). Hence \( T(n)=\Theta(n \log n) \)

2- a) \( T(n) = 2T(n/2) + O(1) \)

Answer: Case 1: \( \Theta(n) \), epsilon=1.

b) \( T(n) = 4T(n/2) + n^2 \)

Answer: Case 2: \( T(n) = (n^2 \log n) \)

3- a) What is the big O complexity of \( T(n)=k^n + \log^8 (n^2 n) + (10n)^k \).

Answer: if \( c<=1 \), \( O(n \log^8 (n)) \), otherwise \( O(k^n) \).

b) \( T(n) = 3T(2\lceil n/3 \rceil) + 1000 \), Stuoge sort..

\( \log_{3/2}(3) = \log_{3}1/\log(3/2) = \log_{3}/(\log_{3}1/2) = -2.72 \)

\( \log_{3/2}(2*1.5) = \log_{3/2}(2) + \log_{3/2}(1.5) = \log_{3/2}(2)+1 = -1.7+1.. \)

Case 1: what is the meaning of this..

4- a) \( M(n) = M(n-1) + 1 \), \( n>0 \), \( M(0) = 0 \), solve by using substation method.

Answer sums to all the ones to \( n \), therefore it is \( \Theta(n) \)

b) \( A(2^k) = A(2^{k-1})+1 \),

Master theorem with no doubt yields with no doubt the result of case 2, and giving a \( \Theta(\log n) \).

5- a) Recursion in d-dimensional mesh (recursion on d-dimensional mesh, \( d>0 \)) \( T(n) = T(n/(2^d)) + 47d^2n^{1/d} \)

Answer: \( h.s =1 \), \( T(n)=\theta(47d^2n^{1/d}) \), \( T(n)=\theta(47d^2n^{1/d}) \), case 3. No need to check regularity.

b) \( T(n) = T(n/2) + \log n \) (PRAM mergesort)

Answer: \( h.s =1 \), for sufficiently large \( n \), \( (\log n)>1 \), appealing to be case 3 and yielding \( T(n) = \log n \), regularity check \( af(n/b) \leq cf(n) \), \( \log(n/2) \leq c\log(n) \), \( (1-c)\log n \leq 1 \), not possible always to satisfy this relation for large \( n \), and \( 0<c<1 \), (for the regularity condition \( 0<c<1 \)). Therefore one needs to use other methods, simply check the equation given below (the final equation obtained at the bottom of iterative tree method), there the first part is 1, and the second part is \( \log^2 n \).

Therefore the result is \( \theta(\log^2 n) \).

\[
T(n) = \Theta(n^{\log_{b}a}) + \sum_{i=0}^{\left\lfloor \log_{b}n \right\rfloor} a f(n/b^i) = \Theta(1) + \sum_{i=0}^{\left\lfloor \log_{b}n \right\rfloor} \log(n/b^i)
\]

\[
= \log n \sum_{i=0}^{\left\lfloor \log_{b}n \right\rfloor} \log(n/2^i), \log n-j, \log n-1, \log n-2, \log n-3, ..., \log n-\log n+1
\]

\[
= \log n \sum_{i=0}^{\left\lfloor \log_{b}n \right\rfloor} \log n-j, (n(n+1)/2), (\log(n)-1)(\log n)/2 \Rightarrow \Theta(\log^2 n)
\]