1. Determine concavity and x-values where the points of inflection occur for
   \( y = -2x^3 + 2x^2 - 7x + 9 \). (do not sketch the graph)

Answer:
\[
y' = -6x^2 + 4x - 7
\]
\[
y'' = -12x + 4 = 0 \quad \Rightarrow \quad x = \frac{1}{3}
\]
is the inflection point.

\[
\begin{array}{c|ccc}
  x & x < \frac{1}{3} & \frac{1}{3} & x > \frac{1}{3} \\
  y'' & y'' > 0 & 0 & y'' < 0 \\
  y & \text{Concave Up} & \text{Inflection point} & \text{Concave Down}
\end{array}
\]

2. For a manufacturer's product, the revenue function is given by
   \( r = 240q + 57q^2 - q^3 \).
   Determine the output for maximum revenue.

Answer:
\[
r' = 240 + 114q - 3q^2 = -(3q^2 - 114q - 240)
\]
\[
= -(3q + 6)(q - 40) = 0 \quad \Rightarrow \quad q = -2 \quad \text{or} \quad q = 40
\]

\( q \) is a quantity so that it cannot be negative. So the extremum point is just 40 corresponding to the maximum from the first derivative test as below:

\[
\begin{array}{c|ccc}
  x & 40 \\
  y' & - & - & + & + \\
  y & \uparrow & \quad \quad \text{increasing} & \quad \quad \text{max} & \quad \quad \text{decreasing}
\end{array}
\]

Therefore the output for maximum revenue is
\[
r\bigg|_{q=40} = 240 \times 40 + 57 \times 40^2 - 40^3 = 9,600 + 91,200 - 64,000 = 36,800
\]
3. Find $\frac{dy}{dx}$ by using logarithmic differentiation for $y = \frac{x(1 + x^2)^2}{\sqrt{2 + x^2}}$

Answer:

$$\ln y = \ln x + 2 \ln (1 + x^2) - \frac{1}{2} \ln (2 + x^2)$$

$$\frac{y'}{y} = \frac{1}{x} + 2 \cdot \frac{2x}{1 + x^2} - \frac{1}{2} \cdot \frac{2x}{2 + x^2}$$

$$y' = \frac{x (1 + x^2)^2}{\sqrt{2 + x^2}} \left[ \frac{1}{x} + \frac{4x}{1 + x^2} - \frac{x}{2 + x^2} \right]$$

$$y' = \frac{2 + x^2}{x} \cdot \frac{1}{2 + x^2} \cdot \frac{1}{x^2} \cdot \frac{1}{2 + x^2}$$

4. Find $\frac{dy}{dx}$ by implicit differentiation for $x^2 e^y + y = 13$

Answer:

$$2xe^y + x^2 e^y y' + y' = 0 \Rightarrow y'(1 + x^2 e^y) = -2xe^y \Rightarrow y' = -\frac{2xe^y}{1 + x^2 e^y}$$

5. Differentiate $y = x^5 e - 5^x$

Answer:

$$y' = 5xe^{5e-1} - \ln 5 \cdot 5^x$$
6. Differentiate $y = \ln^2(2x + 11)$

Answer:

$$y' = 2\ln(2x+11)\frac{2}{2x+11} = \frac{4\ln(2x+11)}{2x+11}$$

7. Find the following limits if they exist.

a) $$\lim_{x \to -\infty} \frac{1+x^2}{(2-3x)^2} = \lim_{x \to -\infty} \frac{x^2}{(-3x)^2} = \lim_{x \to -\infty} \frac{x^2}{9x^2} = \lim_{x \to -\infty} \frac{1}{9} = \frac{1}{9}$$

b) $$\lim_{t \to -2} \frac{t^2 - 4}{t+2} = \lim_{t \to -2} \frac{(t-2)(t+2)}{t+2} = \lim_{t \to -2} (t-2) = -4$$

8. Find the points of discontinuity for $f(x) = \begin{cases} 
    x + 4 & \text{if } x > -2 \\
    3x + 6 & \text{if } x \leq -2 
\end{cases}$

Answer:

i. $f(x)$ is defined for all the real numbers.

ii. $$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (x+4) = 2$$
$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (3x+6) = 0$$

$$\Rightarrow \lim_{x \to -2^+} f(x) \neq \lim_{x \to -2^-} f(x)$$

So there is no limit at $x = -2$ which is a point of discontinuity.
9. Find the present value of $4000 due in 12 years at 7% compounded semi-annually. (write equation only)

Answer:

\[ P = S \left( 1 + \frac{r}{n} \right)^{-rn} = $4000 \left( 1 + \frac{0.07}{2} \right)^{-12 \times 2} = $4000 (1.035)^{-24} \]

10. a) \( 2 + (2)7^{2x+3} = 30 \): Find \( x \) in terms of \( \ln 2 \) and \( \ln 7 \).

Answer:

\[ 7^{2x+3} = \frac{30 - 2}{2} = 14 \Rightarrow (2x + 3)\ln 7 = \ln 14 = \ln 7 + \ln 2 \]

\[ (2x + 3 - 1)\ln 7 = \ln 2 \Rightarrow x = \frac{1}{2} \left( -2 + \frac{\ln 2}{\ln 7} \right) = -1 + \frac{\ln 2}{2\ln 7} \]

b) Find \( x \): \( 2 \log x = \log 4 + \log(x - 1) \)

Answer:

\[ \log x^2 = \log (4(x - 1)) \Rightarrow x^2 = 4x - 4 \]

\[ \Rightarrow x^2 - 4x + 4 = (x - 2)^2 = 0 \]

\[ \Rightarrow x = 2 \]