1) A company management would like to know the total sales units that are required for the company to earn a profit of $10,000. The following data are available: unit selling price of $20; variable cost per unit of $15; total fixed cost of $60,000. Determine the required sales units.

**Solution:**

Profit: \( P = 10,000 \)

Unit Selling Price: \( S = 20 \)

Variable Cost per Unit: \( VC = 15 \)

Total Fixed Cost: \( FC = 60,000 \)

Determine the required sales units?

Let \( u \) denote sales unit

Total Costs: \( TC = VC + FC = 15u + 60,000 \)

Total Sales: \( TS = 20u \)

\[ P = TS - TC = 20u - 15u - 60,000 = 10,000 \Rightarrow 5u = 70,000 \]

\[ → u = 14,000 \]

2) A company manufacturer a product that has a unit selling price of $30 and a unit cost of $20. If fixed costs are $30,000, determine the least number of units that must be sold for the company to have a profit.

**Solution:**

Unit Selling Price: \( S = 30 \)

Variable Unit Cost: \( VC = 20 \)

Fixed Cost: \( FC = 30,000 \)

Determine the least number of units sold to have a profit.

Let \( u \) denote sales unit

Total Costs: \( TC = VC + FC = 20u + 30,000 \)

Total Sales: \( TS = 30u \)

Profit: \( P = TS - TC = 30u - (20u + 30,000) = 10u - 30,000 > 0 \) to have profit

\[ 10u > 30,000 \Rightarrow u > 3,000 \]

The least number of units to be sold is more than 3,000 to have a profit.

3) Find the domain and range the following functions:

a) \( y = f(x) = \frac{1-x}{\sqrt{x^2 - x - 3/4}} \)

b) \( y = f(x) = \frac{3x^2 + 2}{\sqrt{x^2 - 3}} \)

c) \( y = f(x) = \frac{3x^2 + 2}{x^2 + 1} \)

d) \( y = f(x) = \frac{x^2 - 3x + 1}{x^2 - 9} \)

**Solution:**

a. \( y = f(x) = \frac{1-x}{\sqrt{x^2 - x - 3/4}} \)
To be definable, the inside of the square root must be greater than zero. So
\[ x^2 - x - \frac{3}{4} > 0 \quad \Rightarrow \quad (x + \frac{1}{2})(x - \frac{3}{2}) > 0 \]

At \( x = -\frac{1}{2} \) and \( x = \frac{3}{2} \), the denominator is zero, so undefined.

<table>
<thead>
<tr>
<th>Roots</th>
<th>-1/2</th>
<th>3/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + \frac{1}{2} )</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>( x - \frac{3}{2} )</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>( (x + \frac{1}{2})(x - \frac{3}{2}) )</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

From the table only the parts in positive signs give the domain than makes nonnegative in the square root. So

**Domain:** \((-\infty, -\frac{1}{2}) \cup (\frac{3}{2}, \infty)\)  \quad **Forbidden Range:** \([-\frac{1}{2}, \frac{3}{2}]\)

To find the range we have to find function values at the domain. First let’s find the function values at the intervals of the domain.

\[ x \to -\infty, \quad y = f(x) = \frac{\sqrt{\frac{1}{x} - 1}}{\sqrt{1 - \frac{1}{x} - \frac{3}{4x^2}}} \to +1 \]

\[ x \to -\frac{1}{2}, \quad y = f(x) = \frac{1-x}{\sqrt{x^2 - x - \frac{3}{4}}} \to \frac{1-\frac{1}{2}}{0} \to \frac{3/2}{0} \to +\infty \]

\[ x \to \frac{3}{2}, \quad y = f(x) = \frac{1-x}{\sqrt{x^2 - x - \frac{3}{4}}} \to \frac{1-\frac{3}{2}}{0} \to \frac{-1/2}{0} \to -\infty \]

\[ x \to \infty, \quad y = f(x) = \frac{\sqrt{\frac{1}{x} - 1}}{\sqrt{1 - \frac{1}{x} - \frac{3}{4x^2}}} \to -1 \]

**Range:** \((-\infty, -1) \cup (1, \infty)\)
4) Using the absolute value symbol, express each fact.
   a) $X$ is between -3 and 3, but is not equal to 3 or -3.
   b) The number $x$ of hours that a machine will operate efficiently from 255 by less than 6
   c) The average monthly income $x$ (in dollars) of a family differs 1050 by less than 120
   d) $x+4$ is less than 5 units from 0.
   e) The distance between 7 and $x$ is 4.

Solution:

Using the absolute value symbol, express each fact.
   f) $x$ is between -3 and 3, but is not equal to 3 or -3.

$$-3 < x < 3 \quad \Rightarrow \quad |x| < 3$$

   g) The number $x$ of hours that a machine will operate efficiently from 255 by less than 6

$$|x - 255| < 6$$

   h) The average monthly income $x$ (in dollars) of a family differs 1050 by less than 120

$$|x - 1050| < 120$$

   i) $x+4$ is less than 5 units from 0.

$$|x + 4| < 5$$

   j) The distance between 7 and $x$ is 4.

It can be $7 - x = 4 \quad \Rightarrow \quad x = 3$

or $x - 7 = 4 \quad \Rightarrow \quad x = 11$

$$|x - 7| = 4 \quad \text{or} \quad |7 - x| = 4$$

supplies the above results.

5) In functions is $y$ a function of $x$? Is $x$ a function of $y$?
   a) $x^2 + y = 0$
   b) $y = 7x^2$
   c) $x^2 + y^2 = 1$

Solution:

a) $y = -x^2$, $y$ is a function of $x$.

b) $y = 7x^2$, $y$ is a function of $x$.

c) $x^2 + y^2 = 1$, Neither $y$ nor $x$ is a function of $x$ or $y$.

6) Solve the following inequalities:
   a) $2x - (7 + x) \leq x$
   b) $3p(1 - p) > 3(2 + p) - 3p^2$
   c) $4 < \left| \frac{2}{3}x + 5 \right|$
   d) $\frac{3y - 1}{3} > \frac{5(y + 1)}{4}$

Solution:

a) $2x - (7 + x) \leq x$

$$2x - 7 - x \leq 0 \quad \Rightarrow \quad -7 \leq 0 \quad \text{true}$$
Solution: all reel numbers

\[ b. \quad 3p(1-p) > 3(2+p) - 3p^2 \]
\[ 3p - 3p^2 > 6 + 3p - 3p^2 \quad \Rightarrow \quad 0 > 6 \quad \text{false} \]

no solution exist for this problem.

c. \[ 4 < \left| \frac{2}{3}x + 5 \right| \]
\[ 4 < \frac{2}{3}x + 5 \quad \Rightarrow \quad -1 < \frac{2}{3}x \quad \Rightarrow \quad -\frac{3}{2} < x \quad \left( \text{or } x > -\frac{3}{2} \right) \]
\[ -4 > \frac{2}{3}x + 5 \quad \Rightarrow \quad -9 > \frac{2}{3}x \quad \Rightarrow \quad -\frac{27}{2} > x \quad \left( \text{or } x < -\frac{27}{2} \right) \]
Solution: \((-\infty, -27/2) \cup (-3/2, +\infty)\)

d. \[ \frac{3y-1}{3} > \frac{5(y+1)}{4} \]
\[ \frac{3y-1}{3} > \frac{5(y+1)}{4} \quad \Rightarrow \quad 12y - 4 > 15y + 15 \]
\[ \Rightarrow \quad 12y - 15y > 15 + 4 \quad \Rightarrow \quad -3y > 19 \]
\[ \Rightarrow \quad y < -\frac{19}{3} \]
Solution: \((-\infty, -19/3)\)

7) A manufacturer sells a product at $8 per unit, selling all produced. The fixed cost is $2,000 and the variable cost is $7 per unit.

a) At what level of production there will be a profit of $4,000.

b) At what level of production there will be a loss of $1,000.

**Solution:**

\( u: \) the number unit produced
\( s = $8 \) (selling price)
\( FC = $2,000 \)
\( VC \) per unit = $7
Total Cost: \( TC = FC + VC = 2,000 + 7u \)
Total Sales: \( TS = 8u \)

\[ a. \quad TS-TC = 4,000 \quad \Rightarrow \quad 8u - (2,000 + 7u) = 4,000 \]
\[ \Rightarrow \quad u = 6,000 \text{ units must be sold.} \]

\[ b. \quad TS-TC = -1,000 \quad \Rightarrow \quad 8u - (2,000 + 7u) = -1,000 \]
\[ \Rightarrow \quad u = 1,000 \text{ units will be sold.} \]
8) If \( f(x) = 2x \) and \( g(x) = 6 + x \), find the following
   a) \((f \circ g)(x)\)  
   b) \((g \circ f)(x)\)  
   c) \((g \circ f)(2)\)

Solution:
   a) \((f \circ g)(x) = f(g(x)) = f(6 + x) = f(u) = 2u = 2(6 + x) = 2x + 12\)
   b) \((g \circ f)(x) = g(f(x)) = g(2x) = g(u) = 6 + u = 6 + 2x\)
   c) \((g \circ f)(2) = 6 + 2.2 = 10\)

9) Determine the x- and y-intercepts of the following functions. Graph them and give the domain and range of each function.  
   a) \(y = 4 - x\)  
   b) \(y = 4 - x^2\)

Solution:
   a) \(y = 4 - x\)

   - x-intercept \( \rightarrow \) Give \( y = 0 \), then find \( x \) \( \rightarrow x = 4 - y = 4; \) x-intercept is \((0, 4)\)
   - y-intercept \( \rightarrow \) Give \( x = 0 \), then find \( y \) \( \rightarrow y = 4 - x = 4; \) y-intercept is \((4, 0)\)

   Since \( y = 4 - x \) is actually represents a line its range and domain has no restrictions
   Therefore
   Range = \((-\infty, +\infty)\)  
   Domain = \((-\infty, +\infty)\)

   ![Graph of y = 4 - x]

   b) \(y = 4 - x^2\)

   - x-intercept \( \rightarrow y = 0 \), then \( 0 = 4 - x^2 \rightarrow x = \pm 2 \) \( \rightarrow (-2, 0) \ and \ (2, 0)\)
   - y-intercept \( \rightarrow x = 0 \), then \( y = 4 \) \( \rightarrow (0, 4)\)

   When you attempt to sketch a graph of a quadratic function which is a parabola, first thing to do is to determine whether it is “upward opening or downward opening type”. To do this we look at the sign of the coefficient of \( x^2 \) term. Here it is “-”, so our parabola is “downward opening.” Second thing to do is find x-intercept and y-intercept points. Third to find “vertex” position which is given as

   \[ x_{\text{vertex}} = \frac{-b}{2a} = \frac{0}{2} = 0 \]
   then to find \( y_{\text{vertex}} \) simply use \( x_{\text{vertex}} \) in the equation

   \[ y_{\text{vertex}} = y(x_{\text{vertex}}) = 4 - (x_{\text{vertex}})^2 = 4 \]

   \((x_{\text{vertex}}, y_{\text{vertex}}) = (0, 4)\)

   The graph is shown above (on the right hand side).
10) Find the x- and y-intercepts of the following functions. Also test for symmetry about the x-axis, the y-axis, and the origin. a) \( y = f(x) = 2x^3 - 8x \)  \( \frac{\partial}{\partial x} y = 0 \) 
\( \rightarrow 0 = 2x^3 - 8x = 2x(x^2 - 4) = 2x(x - 2)(x + 2) \)  \( \rightarrow x = 0, \pm 2 \)

**Solution:**

a) 
\( y = f(x) = 2x^3 - 8x \)

\( x \)-intercepts \( \rightarrow y = 0 \)  \( \rightarrow 0 = 2x^3 - 8x = 2x(x^2 - 4) = 2x(x - 2)(x + 2) \)  \( \rightarrow x = 0, \pm 2 \)

There are 3 points for x-intercepts. (0,0); (2,0); (-2,0)

\( y \)-intercept \( \rightarrow x = 0 \)  \( \rightarrow y = 2.0 - 8.0 = 0 \)  \( \rightarrow (0,0) \)

\( y \)-axis symmetry \( \rightarrow (-a,b) \leftrightarrow (a,b) \) so for x-axis symmetry \( f(x) = f(-x) \)

\( f(-a) = 2.(-a)^3 - 8(-a) = -2a^3 + 8a \)

\( \neq f(a) \) \( \rightarrow \) NOT y-axis symmetry

\( x \)-axis symmetry \( \rightarrow (a,-b) \leftrightarrow (a,b) \)

Let us try \( x = 1 \)  \( \rightarrow y = -6 \) if \( (1,-6) \) is a point then \( (1,6) \) MUST also be a point on the graph.

\( x = -1 \)  \( \rightarrow y = +6 \)  \( \rightarrow \) Therefore NOT x-axis symmetry

symmetry about origin requires \( (a,b) \rightarrow (-a,-b) \)

We found that \( (1,-6) \leftrightarrow (-1,6) \)  \( \rightarrow \) Therefore It is symmetric about origin.

11) Find the equation of the straight line that has the following properties:

a) Passes through \((4, -2)\) and \((-6, 3)\)

b) Passes through \((-2, 5)\) and has a slope 4

c) Perpendicular to \( y = x + 5 \) and passes through \((1, 1)\)

**Solution:**

a) slope \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{-6 - 4} = \frac{5}{-10} = -0.5 \)

\( y - y_1 = m(x - x_1) = -0.5(x - 4) = -0.5x + 2 = y - (-2) \)  \( \rightarrow y = -0.5x \)

c) For perpendicular lines, their slopes must satisfy the condition \( m_1m_2 = -1 \)

\( m_1 = +1 \)  \( \rightarrow m_2 = -1 \)  \( \rightarrow \) and the point is \((1,1)\)

\( y - y_1 = m(x - x_1) = -1(x - 1) \)  \( \rightarrow y = -x + 2 \)

12) For the following find a) the vertex b) x-intercepts, c) y-intercept, d) sketch the graph.

a) \( y = 12 - 8s + s^2 \)  \( \frac{\partial}{\partial s} y = 0 \)

b) \( y = x^2 - 4 \)

c) \( y = -4x^2 \)

**Solution:**

a) 
\( y = 12 - 8s + s^2 = s^2 - 8s + 12 \)

\( s_{\text{vertex}} = \frac{b}{2a} = \frac{-(-8)}{2} = 4 \)  \( \rightarrow y_{\text{vertex}} = 4^2 - 8.4 + 12 = -4 \)

\( s \)-intercepts 

\( y = 0 \)  \( 0 = s^2 - 8s + 12 \)  \( \rightarrow s = +2, +6 \)
13) The demand function for an electronic company’s computer line is \( p = 1200 - 3q \), where \( p \) is the price per unit when \( q \) units are demanded by consumers. Find the level of production that will maximize the manufacturer’s total revenue, and determine this revenue.

**Solution:**

Total Revenue = \( p \cdot q = (1200 - 3q)q = -3q^2 + 1200q \)

Maximum revenue would occur at the vertex of the parabola. Therefore

\[ q_{\text{vertex}} = \frac{-b}{2a} = \frac{-1200}{2(-3)} = 200 \text{ units} \]

Maximum revenue at this production level is

\[ r_{\text{vertex}} = -3(200)^2 + 1200 \cdot 200 = 120,000 \]

14) Suppose consumers will demand 40 units a product when the price is $12 per unit and 26 units when the price is $19 each. Find the demand equation assuming that it is linear. Find the price per unit when 30 units are demanded.

**Solution:**

Two points are

\((q, p) = (12, 40)\)

\(= (19, 26)\)

The equation of the line passing through two points

\[ m = \frac{p_2 - p_1}{q_2 - q_1} = \frac{40 - 26}{12 - 19} = -2 \]

\[ p - p_1 = m(q - q_1) = -2(q - 12) = p - 40 \quad \rightarrow \quad p = -2q + 64 \]