MATH 172 - PROBLEM SET 4

1. Sketch the given surfaces.
   a) \(2x + 3y + z = 9\) 
   b) \(z = x\)

2. Find the indicated partial derivatives.
   a) \(P = l^3 + k^3 - lk; \quad \frac{\partial P}{\partial l}, \frac{\partial P}{\partial k}\) 
   b) \(z = \frac{x}{x + y}; \quad \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\) 
   c) \(f(x, y) = \ln(\sqrt{x^2 + y^2}); \quad \frac{\partial}{\partial y}[f(x, y)]\) 
   d) \(w = e^{x^3y}; \quad w_{xy} = (x, y, z)\) 
   e) \(f(x, y) = xy \ln(xy); \quad f_{xy}(x, y)\) 
   f) \(w = e^{x+y+z} \ln xyz; \quad \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial z \partial x}\)

3. If \(f(x, y, z) = \frac{x + y}{xz}\) find \(f_{xyz}(2,7,4)\)

4. If \(f(x, y, z) = (6x + 1)e^{y^3 \ln(z+1)}\) find \(f_{xyz}(0,1,0)\)

5. If \(w = x^2 + 2xy + 3y^2, \quad x = e^r, \quad \text{and} \quad y = \ln(r + s), \text{find} \ \frac{\partial w}{\partial r} \text{and} \ \frac{\partial w}{\partial s}\)

6. If \(z^2 + \ln(yz) + \ln(z + x + z) = 0, \text{find} \ \frac{\partial z}{\partial y}\)

7. If a manufacturer's production function is defined by \(P = 20l^{0.7}k^{0.3}\), determine the marginal productivity functions.

8. A manufacturer's cost for producing \(x\) units of product X and \(y\) units of product Y is given by \(c = 3x + 0.05xy + 9y + 500\), determine the (partial) marginal cost with respect to \(x\) when \(x=50\) and \(y=100\).

9. Examine \(f(x, y) = x^2 + 2y^2 - 2xy - 4y + 3\) for relative extrema.

10. A company manufactures two products, X and Y, and the joint-cost function for these products is given by \(c = 0.002(x+y)^2 + x + 0.25y + 8000\), where c is the total cost of producing \(x\) units of X and \(y\) units of Y. Determine the marginal cost with respect to \(x\) when \(x=450\) and \(y=550\).

11. A company's production function is given by \(500 - 3x - 2y - 10\), where \(c\) is the total cost of producing \(x\) units of X and \(y\) units of capital. Determine: (a) the marginal production function with respect to \(L\), (b) the marginal production function with respect to \(k\).

12. The demand function for product A is \(q_A = 100 \frac{\partial p_A}{p_A}\), and the demand function for product B is \(q_B = 20 + 3p_A - 2p_B\), where \(q_A\) and \(q_B\) are the quantities demanded for A and B, respectively, and \(p_A\) and \(p_B\) are their respective prices. Determine: (a) the marginal demand for A with respect to \(p_B\), (b) the marginal demand for B with respect to \(p_A\), (c) whether A and B are competitive, complementary, or neither.

13. Let \(q_A = 50 - 5p_A + 6p_B^2\) and \(q_B = 20\sqrt{p_A} \over p_A^2\) be demand functions, where \(p_A\) and \(p_B\) are prices for products A and B, respectively. Find all four marginal demand functions.

14. Use implicit partial differentiation to find \(\frac{\partial z}{\partial y}\) from \(e^{xy} + 7x^3 + 8z - 19 = 0\)

15. For \(\ln(xyz) + e = e^y + 1\), the partial derivative \(\frac{\partial z}{\partial x}\) evaluated at \(x = e^{-2}, y = 1, z = e^3\)

16. If \(f(x, y) = 2x^4y^3 - 3x^2y^3 + 4xy - x + 2y + 4\), find: \(f_x(x, y), f_y(x, y), f_{xy}(x, y), f_{yy}(x, y), f_{yx}(-1,1)\)

17. Let \(f(x, y, z) = \ln(x^4 + 6y^2) - 2z^3e^{x^3} + x^2y^3\) . Find \(\frac{\partial^3 f}{\partial x \partial y \partial z}\)

18. If \(z = (x^2 + y^2)^{10}\) where \(x = 4r^2s^3\) and \(y = e^{2r+3s-3}\), then by means of the chain rule, (a) find \(\frac{\partial z}{\partial r}\); (b) evaluate \(\frac{\partial z}{\partial s}\) when \(r=0\) and \(s=1\).

19. Determine the critical points of \(f(x, y) = 2xy - 3x - y^2 - 3y^2\) and also determine by the second-derivative test whether each point corresponds to a relative maximum, to a relative minimum, to neither, or whether the test gives no information.

20. A television manufacturing company makes two types of TVs. The cost of producing \(x\) units of type A and \(y\) units of type B is given by the function \(C(x, y) = 120 + x^3 + 8y^3 - 24xy\). How many units of type A and B televisions should the company produce to minimize its cost?