$i = \frac{dQ}{dt}$

integral op-amp
alıcı devre

come in discrete form and tend to be more bulky and expensive. For this reason, inductors are not as versatile as capacitors and resistors, and they are more limited in applications. However, there are several applications in which inductors have no practical substitute. They are routinely used in relays, delays, sensing devices, pick-up heads, telephone circuits, radio and TV receivers, power supplies, electric motors, microphones, and loudspeakers, to mention a few.

Capacitors and inductors possess the following three special properties that make them very useful in electric circuits:

1. The capacity to store energy makes them useful as temporary voltage or current sources. Thus, they can be used for generating a large amount of current or voltage for a short period of time.
2. Capacitors oppose any abrupt change in voltage, while inductors oppose any abrupt change in current. This property makes inductors useful for spark or arc suppression and for converting pulsating dc voltage into relatively smooth dc voltage.
3. Capacitors and inductors are frequency sensitive. This property makes them useful for frequency discrimination.

The first two properties are put to use in dc circuits, while the third one is taken advantage of in ac circuits. We will see how useful these properties are in later chapters. For now, consider three applications involving capacitors and op amps: integrator, differentiator, and analog computer.

$Q = C.V$

$\frac{dQ}{dt} = C\frac{dV}{dt}$

6.6.1 Integrator

Important op amp circuits that use energy-storage elements include integrators and differentiators. These op amp circuits often involve resistors and capacitors; inductors (coils) tend to be more bulky and expensive.

The op amp integrator is used in numerous applications, especially in analog computers, to be discussed in Section 6.6.3.

> An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

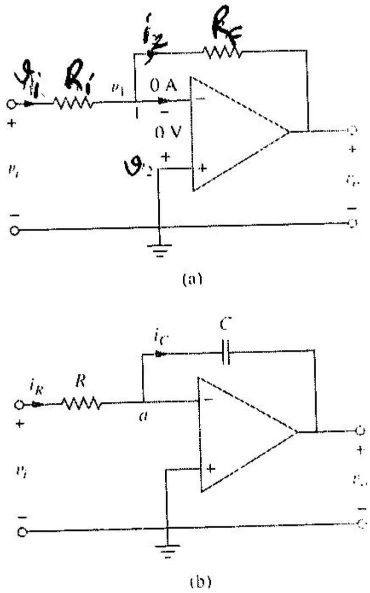


Figure 6.35
Replacing the feedback resistor in the inverting amplifier in (a) produces an integrator in (b).

If the feedback resistor R_f in the familiar inverting amplifier of Fig. 6.35(a) is replaced by a capacitor, we obtain an ideal integrator, as shown in Fig. 6.35(b). It is interesting that we can obtain a mathematical representation of integration this way. At node a in Fig. 6.35(b),

$$i_R = i_C \tag{6.32}$$

$i_R = i_C$

But

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

$i_R = \frac{v_i}{R}$

$i_C = -C\frac{dv_o}{dt}$

Substituting these in Eq. (6.32), we obtain

$$\frac{v_i}{R} = -C\frac{dv_o}{dt} \tag{6.33a}$$

$$dv_o = -\frac{1}{RC}v_i\,dt \tag{6.33b}$$

$\frac{v_i}{R} = -C\frac{dv_o}{dt}$

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau$$

Integrating both sides gives

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau \qquad (6.34)$$

To ensure that $v_o(0) = 0$, it is always necessary to discharge the integrator's capacitor prior to the application of a signal. Assuming $v_o(0) = 0$,

$$\boxed{v_o = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau} \qquad (6.35)$$

$$\boxed{v_o = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau}$$

which shows that the circuit in Fig. 6.35(b) provides an output voltage proportional to the integral of the input. In practice, the op amp integrator requires a feedback resistor to reduce dc gain and prevent saturation. Care must be taken that the op amp operates within the linear range so that it does not saturate.

$R_1C = 3\cdot10^6\cdot2\cdot10^{-6} = 6\,s.$

$R_2C = 100\cdot10^3\cdot2\cdot10^{-6} = 0,2\,s.$

Example 6.13

If $v_1 = 10\cos 2t$ mV and $v_2 = 0.5t$ mV, find v_o in the op amp circuit in Fig. 6.36. Assume that the voltage across the capacitor is initially zero.

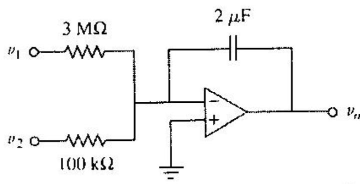


Figure 6.36
For Example 6.13.

$$v_o = -\frac{1}{R_1C}\int_0^t v_1 d\tau - \frac{1}{R_2C}\int_0^t v_2 d\tau$$

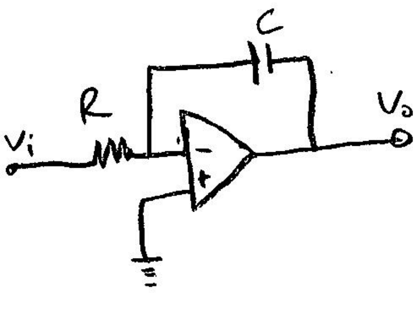
Solution:
This is a summing integrator, and

$$\frac{d}{dt}\left(\frac{1}{2}\sin 2t\right) = \cos 2t$$

$$\begin{aligned}
v_o &= -\frac{1}{R_1C}\int v_1\,dt - \frac{1}{R_2C}\int v_2\,dt \\
&= -\frac{1}{3\times10^6\times2\times10^{-6}}\int_0^t 10\cos(2\tau)d\tau \\
&\quad -\frac{1}{100\times10^3\times2\times10^{-6}}\int_0^t 0.5\tau\,d\tau \\
&= -\frac{1}{6}\frac{10}{2}\sin 2t - \frac{1}{0.2}\frac{0.5t^2}{2} = \underline{-0.833\sin 2t - 1.25t^2 \text{ mV}}
\end{aligned}$$

Practice Problem 6.13

The integrator in Fig. 6.35(b) has $R = 100$ kΩ, $C = 20$ μF. Determine the output voltage when a dc voltage of 2.5 mV is applied at $t = 0$. Assume that the op amp is initially nulled.



Answer: $-1.25t$ mV.

6.6.2 Differentiator

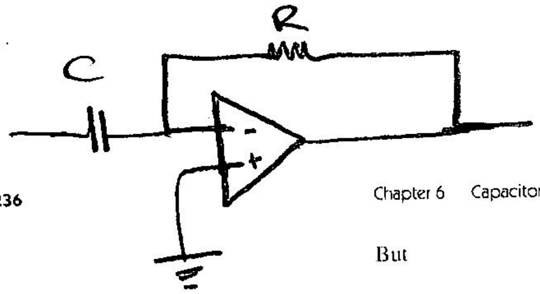
A differentiator is an op amp circuit whose output is proportional to the rate of change of the input signal.

In Fig. 6.35(a), if the input resistor is replaced by a capacitor, the resulting circuit is a differentiator, shown in Fig. 6.37. Applying KCL at node a,

$$i_R = i_C \qquad (6.36)$$

$$-\frac{1}{RC}\int_0^t v_i\,d\tau$$

$$-\frac{1}{20\cdot10^{-6}\cdot10^5}\int_0^t 2,5\,d\tau = -1,25t$$



$i_c = i_R$

$C\dfrac{dv_i}{dt} = -\dfrac{v_o}{R}$

$\boxed{v_o = -RC\dfrac{dv_i}{dt}}$

But

$$i_R = -\frac{v_o}{R}, \qquad i_C = C\frac{dv_i}{dt}$$

Substituting these in Eq. (6.36) yields

$$\boxed{v_o = -RC\frac{dv_i}{dt}} \tag{6.37}$$

showing that the output is the derivative of the input. Differentiator circuits are electronically unstable because any electrical noise within the circuit is exaggerated by the differentiator. For this reason, the differentiator circuit in Fig. 6.37 is not as useful and popular as the integrator. It is seldom used in practice.

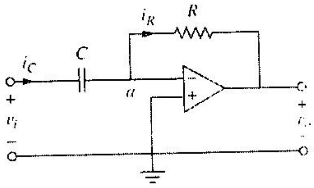


Figure 6.37
An op amp differentiator.

Example 6.14

Sketch the output voltage for the circuit in Fig. 6.38(a), given the input voltage in Fig. 6.38(b). Take $v_o = 0$ at $t = 0$.

Solution:
This is a differentiator with

$$RC = 5 \times 10^3 \times 0.2 \times 10^{-6} = 10^{-3}\text{ s}$$

For $0 < t < 4$ ms, we can express the input voltage in Fig. 6.38(b) as

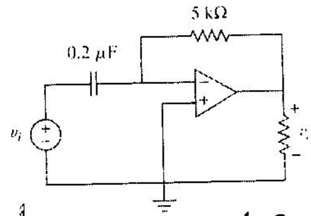
$0<t<2$ ms

$$v_i = \begin{cases} 2000t & 0 < t < 2\text{ ms} \\ 8 - 2000t & 2 < t < 4\text{ ms} \end{cases}$$

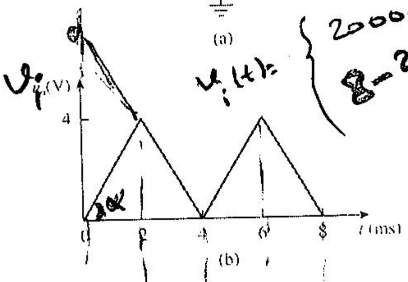
This is repeated for $4 < t < 8$ ms. Using Eq. (6.37), the output is obtained as

$$v_o = -RC\frac{dv_i}{dt} = \begin{cases} -2\text{ V} & 0 < t < 2\text{ ms} \\ 2\text{ V} & 2 < t < 4\text{ ms} \end{cases}$$

Thus, the output is as sketched in Fig. 6.39.



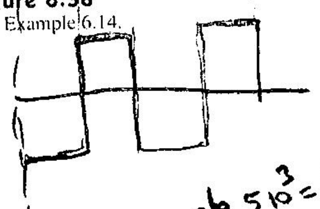
$v_i(t) = \begin{cases} 2000t \\ 8 - 2000t & 2<t<4 \end{cases}$



$\dfrac{4}{2\text{ ms}}$

$2000t$

Figure 6.38
For Example 6.14.



$RC = 0{,}2 \cdot 10^{-6} \cdot 5\,10^{3} = 10^{-3}\text{ s}.$

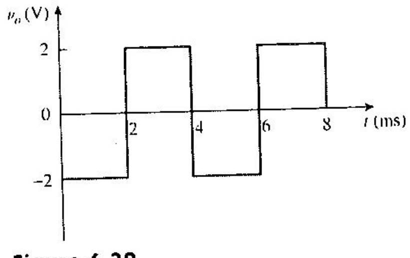


Figure 6.39
Output of the circuit in Fig. 6.38(a).

Practice Problem 6.14

The differentiator in Fig. 6.37 has $R = 100$ kΩ and $C = 0.1$ μF. Given that $v_i = 1.25t$ V, determine the output v_o.

Answer: -12.5 mV.

Chapter 6, Problem 70.

Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_0 = -50\int_0^t v_i(t)\,dt$$

Assume $v_o = 0$ at $t = 0$.

Chapter 6, Solution 70.

One possibility is as follows:

$$\frac{1}{RC} = 50$$

Let R = 100 kΩ, $C = \dfrac{1}{50\times100\times10^3} = 0.2\mu F$

$\frac{1}{RC} = 50$

$\frac{1}{100\,k\Omega\cdot C} = 50\,s$

$C = \frac{1}{100\cdot10^3\cdot50} = 0{,}2\cdot10^{-6}\,F$

Chapter 6, Problem 71.

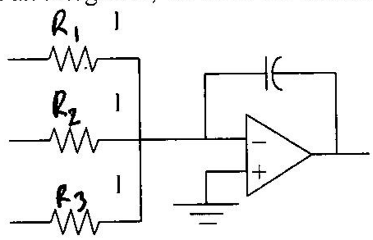
Show how you would use a single op amp to generate

$$v_0 = -\int (v_1 + 4v_2 + 10v_3)\,dt$$

If the integrating capacitor is $C = 2\ \mu F$, obtain other component values.

Chapter 6, Solution 71.

By combining a summer with an integrator, we have the circuit below:



$\frac{1}{R_1C} = 1$, $R_1 = \frac{1}{2\cdot10^{-6}} = 500k$

$\frac{1}{R_2C} = 4$, $R_2 = 125\,k$

$\frac{1}{R_3C} = 10$, $R_3 = 50\,k$

$$v_o = -\frac{1}{R_1C}\int v_1 dt - \frac{1}{R_2C}\int v_2 dt - \frac{1}{R_2C}\int v_2 dt$$

For the given problem, C = **2μF**,

$R_1C = 1 \longrightarrow R_1 = 1/(C) = 10^6/(2) =$ **500 kΩ**

$R_2C = 1/(4) \longrightarrow R_2 = 1/(4C) = 500k\Omega/(4) =$ **125 kΩ**

$R_3C = 1/(10) \longrightarrow R_3 = 1/(10C) =$ **50 kΩ**

Chapter 6, Problem 69.

An op amp integrator with $R = 4\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$ has the input waveform shown in Fig. 6.88. Plot the output waveform.

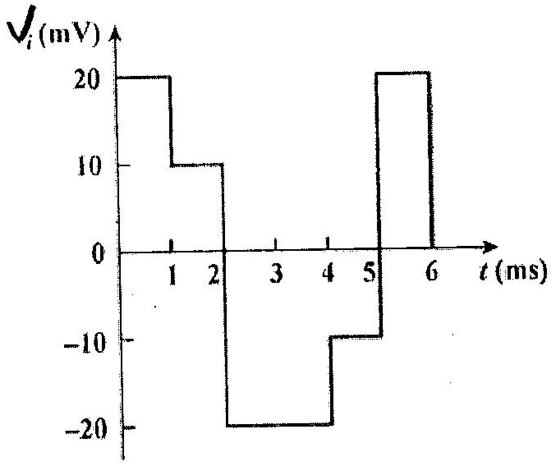


Figure 6.88

$$RC = 4$$

$$v_o = -\frac{1}{RC}\int_0 v_i\, d\tau$$

$$0 < t < 1;\quad -\frac{1}{4}\int_0^t 20\, d\tau = -5t \text{ mV}$$

$$1 < t < 2;\quad -\frac{1}{4}\int_1^t 10\, d\tau + v_o(1) = -2{,}5(t-1) - 5 = -2{,}5t - 2{,}5 \text{ mV}$$

$$2 < t < 4;\quad -\frac{1}{4}\int_2^t -20\, d\tau + v_o(2) = 5\tau\Big|_{\tau=2}^{t} - 7{,}5 = \underline{5t - 17{,}5 \text{ mV}}$$

$$4 < t < 5;\quad -\frac{1}{4}\int_4^t -10\, d\tau + v_o(4) = 2{,}5\tau\Big|_{\tau=4}^{\tau=t} + 2{,}5 = (2{,}5t - 7{,}5) \text{ mV}$$

$$5 < t < 6;\quad -\frac{1}{4}\int_5^t 20\, d\tau + v_o(5) = -5\tau\Big|_{\tau=5}^{\tau=t} + 5 = -5t + 30 \text{ mV}$$

Chapter 6, Solution 69.

$RC = 4 \times 10^6 \times 1 \times 10^{-6} = 4$

$$v_o = -\frac{1}{RC}\int v_i dt = -\frac{1}{4}\int v_i dt$$

For $0 < t < 1$, $v_i = 20$, $v_o = -\frac{1}{4}\int_0^t 20dt = -5t$ mV

For $1 < t < 2$, $v_i = 10$, $v_o = -\frac{1}{4}\int_1^t 10dt + v(1) = -2.5(t-1) - 5$

$= -2.5t - 2.5$mV

For $2 < t < 4$, $v_i = -20$, $v_o = +\frac{1}{4}\int_2^t 20dt + v(2) = 5(t-2) - 7.5$

$= 5t - 17.5$ mV

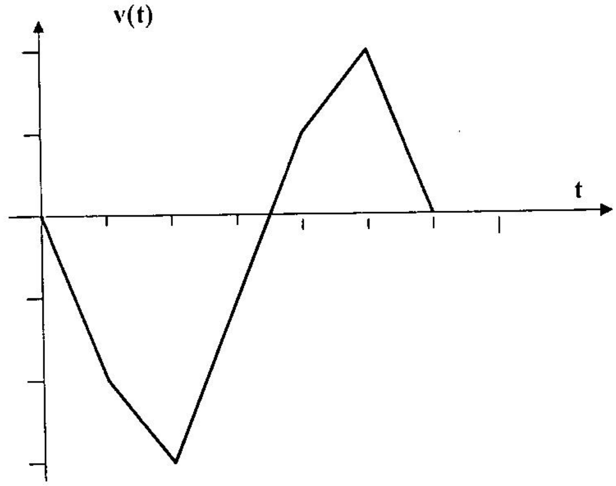
For $4 < t < 5$m, $v_i = -10$. $v_o = \frac{1}{4}\int_4^t 10dt + v(4) = 2.5(t-4) + 2.5$

$= 2.5t - 7.5$ mV

For $5 < t < 6$, $v_i = 20$. $v_o = -\frac{1}{4}\int_5^t 20dt + v(5) = -5(t-5) + 5$

$= -5t + 30$ mV

Thus $v_o(t)$ is as shown below:



Chapter 6, Problem 72.

At t = 1.5 ms, calculate v_o due to the cascaded integrators in Fig. 6.89. Assume that the integrators are reset to 0 V at t = 0.

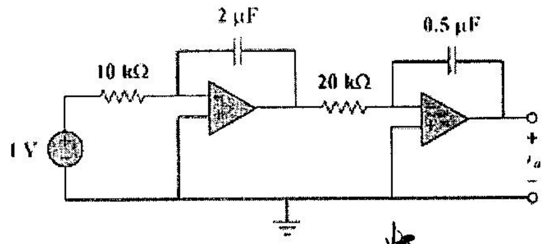


Figure 6.89

$-\frac{1}{10k.2.10^{-6}} \int_0^{t} 1\, d\tau = -50 . t . 10^{-3}\ V$

$= -0,05t$ mV

$-\frac{1}{20k.0,5.10^{-6}} \int_0^{1,5} -0,05t\, d\tau$

$5 \frac{t^2}{2} \Big|_0^{1,5} = \frac{5.2,25}{2}$

$= 5,625$ mV

Chapter 6, Solution 72.

The output of the first op amp is

$$v_1 = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{10x10^3 x2x10^{-6}}\int_0^t v_i dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{20x10^3 x0.5x10^{-6}}\int_0^t (-50t)dt$$

$$= 2500t^2$$

At t = 1.5ms,

$v_o = 2500(1.5)^2 x10^{-6}$ = **5.625 mV**