Modeling of Harmonic Control Arrays using MATLAB/Simulink with an Application to a Hot Rolling Mill Process

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***Abstract*— It was recently demonstrated that the Harmonic Control Array (HCA) method is a successful control strategy for systems with periodic references or disturbances. To construct the required control signal to achieve zero steady-state error, the HCA appropriately modifies the complex levels of the harmonic components of the system input. In a real-time application, a discrete-time implementation is required for the method to be applied through a digital device since the signals and parameters involved are complex-valued. An efficient MATLAB/Simulink modeling of HCAs is explained step-by-step in this paper with an implementation on a typical test system. Then, the HCA performance is compared with an internal model control application on a hot rolling mill process.**

I. INTRODUCTION

The Harmonic Control Array (HCA) method has been developed for control systems with periodic reference or disturbance signals. The HCA framework provides flawless periodic reference tracking and disturbance rejection by automatically constructing the compensating periodic control signal. The HCA uses the running Fourier series integral in its complex nature to efficiently retrieve the harmonic components of the real-time error signal in a unitary feedback system structure. Then these complex signals for each harmonic frequency are controlled separately by a controller working in the complex domain and in real-time. Here many alternative control techniques can be employed to build the internal structure of the HCA. For the current work, we use a variety of PI controllers with complex gain coefficients. This relatively simple structure is generally efficient for periodic harmonic distortion correction or periodic tracking. Like the famous PID controller mostly preferred for constant reference regulation problems, the HCA with internal PI controllers can achieve similar success with a simple structure for the control problems with periodic references or disturbances. After the complex control signals are constructed for each harmonic frequency, they are assembled using the Fourier synthesis operation to obtain the real-time control signal to be applied to the controlled plant.

Initial concepts of the Harmonic Control Array method were introduced in [1] and [2]. This innovative technique was introduced in [3] with a real-time application on periodic position control. Then as applications on power electronics, the HCA is applied to single-phase stand-alone inverters [4] and active power filters [5]. A microprocessor implementation procedure of the HCA method is provided in [6] to show its effectiveness and feasibility in real-time applications. Further information may be found in the above references about the description of the HCA method with simulations and real-time applications.

In this paper, a MATLAB/Simulink model is constructed for HCAs in an efficient way. All the internal blocks of an HCA are explained and built using the existing blocks used in the current Simulink model library. A typical test system is considered as an example in this setup, and various complex and real signals produced by the HCA are provided in the figures.

The Internal Model Control (IMC) framework provides a control solution for systems by integrating the dynamical system model of the given signal in the feedback structure. Compensating controllers can be designed under the condition that the closed loop system is stable. The early works on this method were given in [7] and [8]. Repetitive controllers use this principle to track or reject periodic signals with a specific period [9]. Compensation of periodic disturbances on the first order systems with time delay was studied in [10].

A repetitive control solution for the hot rolling mill process is studied in [11]. The attenuation of eccentric roll disturbance is an essential problem in steel manufacturing. Modeling the process as a first order time-delay system, an IMC-based solution has been suggested in [12] recently. Furthermore, [13] studies a similar system considering multi-harmonic periodic disturbance. Here, we consider the same system and compare our results with [12].

II. HARMONIC CONTROL ARRAYS

Consider a typical unitary feedback system where $e(t)$ is the difference between the reference input $r(t)$ and the system output $y(t)$, and $u(t)$ is the system input produced by the controller. When an HCA is chosen as the controller, a certain signal period, T, needs to be specified as the fundamental period. The angular frequency is defined as $\omega = 2\pi/T$. The highest harmonic number considered is H. The internal blocks of an HCA are as follows.

Disperser: Using a running Fourier series integral, harmonic components can be obtained from a time domain signal. Here, at any time instant, we consider the last time period of the incoming signal. Therefore we continuously track the changes in the complex levels of the harmonic signals as fast as possible. The dispersed signal of the error for the hth harmonic is obtained as

$$\langle e\rangle_h(t) = \frac{1}{T}\int_{t-T}^{t} e(\tau)e^{-jh\omega\tau}d\tau. \quad (1)$$

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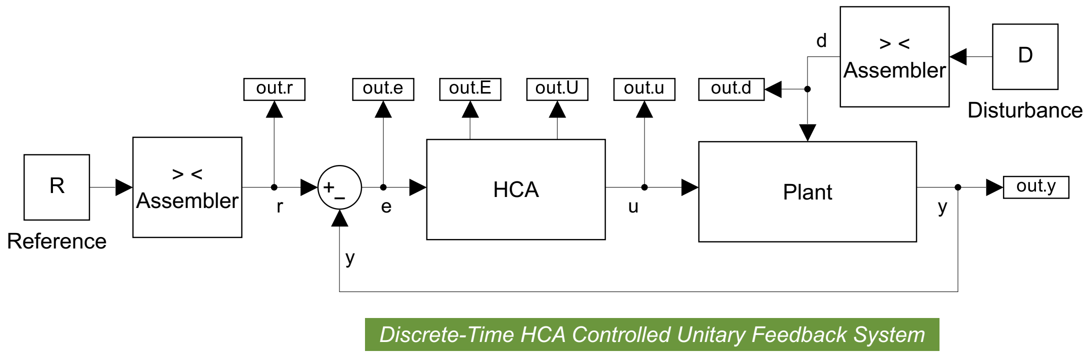

Fig. 1. A unitary feedback system with an HCA controller.

We combine these dispersed signals in the vector form to obtain the dispersion of e as

$$\langle e\rangle = \begin{bmatrix} \langle e\rangle_0 \\ \langle e\rangle_1 \\ \vdots \\ \langle e\rangle_H \end{bmatrix}. \tag{2}$$

It is assumed that $e(t)$ is a real-valued signal, therefore, there is no need to calculate the negative harmonics here. If the signal $e(t)$ is periodic with T, $\langle e\rangle$ will have a complex constant vector value in time.

The discrete-time version of (1) can be given as

$$\langle e\rangle_h[n] = \frac{1}{N}\sum_{k=n-N+1}^{n} e[k]e^{-j2\pi hk/N} \tag{3}$$

where $N = T/T_s$ is the number of samples in the fundamental period, which should be an integer number, and T_s is the sampling time. N/H should be high enough so that the highest harmonic signal is represented well. When especially N is high, the computational burden of (3) may be too much. In this case, we can use the following equivalent calculation for the discrete-time disperser

$$\langle e\rangle_h[n] = \langle e\rangle_h[n-1] + (e[n] - e[n-N])\, e^{-j2\pi hn/N}/N. \tag{4}$$

Once the exponential terms are pre-calculated, (4) only needs one complex multiplication and a memory buffer of N real numbers.

HCA PI Controller: The HCA internal controller treats each harmonic signal obtained from the disperser separately, and tries to construct the control signal harmonics so that the steady state-error asymptotically approaches zero as fast as possible. Normally, to be able to achieve this, integral controllers are needed for each harmonic signal so that even when the error reaches zero, the integral outputs still produce the appropriate complex values to be injected into the system input. To make the transient state shorter, on the other hand, proportional controllers can be employed. Therefore we have the following for the calculation of the dispersion of the system input

$$\langle u\rangle(t) = K_P\langle e\rangle(t) + K_I\int_0^t \langle e\rangle(t)dt. \tag{5}$$

Here, K_P and K_I are complex-valued proportional and integral gain matrices. The off-diagonal or cross terms in these matrices may be useful for nonlinear systems; however, if not used, these matrices could be chosen as diagonal matrices, in which case, every error harmonic signal only modifies the corresponding control harmonic signal. The term *Harmonic Control Array* was coined because an array of controllers for each harmonic is acting to construct the control signal.

The discrete-time version of (5) can be used as

$$\langle u\rangle[n] = K_P\langle e\rangle[n] + K_I T_s\sum_0^n \langle e\rangle[n]. \tag{6}$$

Assembler: The harmonic assembler recombines harmonic components obtained from the HCA internal controller to construct the real-time control signal. Assuming $e(t)$ and $u(t)$ are real valued signals, we can use the Fourier series synthesis to construct the control signal as

$$u(t) = \langle u\rangle_0(t) + 2Re\left\{\sum_{h=1}^{H}\langle u\rangle_h(t)e^{jh\omega t}\right\} \tag{7}$$

where the discrete version can be obtained similarly as

$$u[n] = \langle u\rangle_0[n] + 2Re\left\{\sum_{h=1}^{H}\langle u\rangle_h[n]e^{j2\pi hn/N}\right\}. \tag{8}$$

III. MODELING USING MATLAB/SIMULINK

A unitary feedback system is considered as in Fig. 1. MATLAB version 9.13 and Simulink version 10.6 are used. Each block is explained in the following subsections.

A. HCA Block

The HCA block consists of the disperser, the HCA PI controller, and the assembler, as shown in Fig. 2, and as described in the previous section.

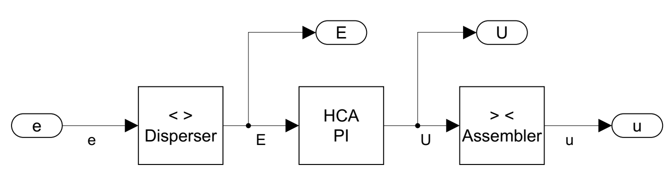

Fig. 2. Blocks in the HCA structure.

Here, the capital letters E and U represent the dispersions of the real-time signals e and u. The HCA block parameters are set in the dialog box as shown in Fig. 3.

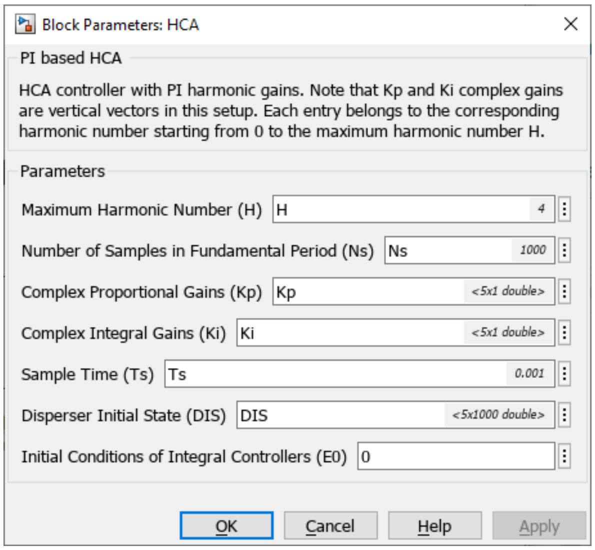

Fig. 3. HCA Block Parameters.

B. Disperser

The disperser process (4) is modeled as shown in Fig. 4. Here, normally, the discrete integrator would be the last block, but for that kind of construction, the numerical errors may be continuously accumulated, diverting the system performance. We use the integral block before so that even if there are errors accumulated in the integral sum, they will be eliminated by the delay and difference block at the end. The Disperser Exponential Sequence block consists of the blocks shown in Fig. 5 where the Direct lookup Table contains the matrix

$$\mathtt{Wd} = \mathtt{exp}(-\mathtt{1j} * 2 * \mathtt{pi} * [0:\mathtt{H}]' * [0:\mathtt{N}-1]/\mathtt{N})/\mathtt{N} \quad (9)$$

which produces the necessary exponential sequence as a complex vector corresponding the current sample number.

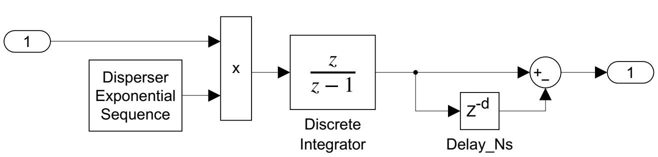

Fig. 4. Construction of the disperser.

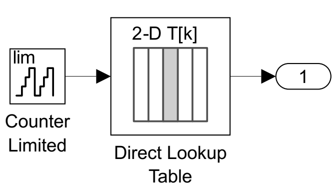

Fig. 5. Construction of the exponential sequence generator.

The Counter Limited recursively produces the current sample number counting from 0 to $N-1$.

C. PI Controllers

The HCA internal controller can be constructed using many different control techniques. Here, we consider PI controllers as shown in Fig. 6. K_P and K_I can be used as square matrices as described in (6). However, if off-diagonal values are not used, K_P and K_I can also be chosen as vectors. In this case, the gain blocks in Fig. 6 should be implemented as element-wise multiplication.

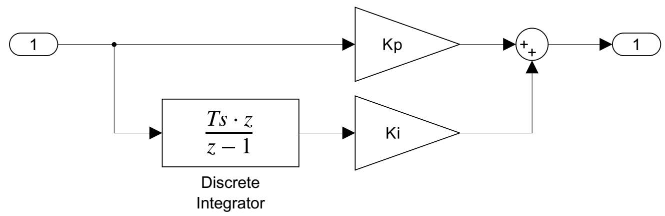

Fig. 6. Construction of the HCA PI controller.

D. Assembler

The assembler block recombines the harmonic vector to construct the control signal as described in (8) and as shown in Fig. 7. Here, the Assembler Exponential Sequence block consists of the blocks shown in Fig. 5 where the Direct Lookup Table contains the matrix

$$\mathtt{Wa} = \mathtt{exp}(\mathtt{1j} * 2 * \mathtt{pi} * [0:\mathtt{H}]' * [0:\mathtt{N}-1]/\mathtt{N})/\mathtt{N}. \quad (10)$$

Here, the gain in the gain block is chosen as [1 2*ones(1,H)] so that (8) is properly calculated.

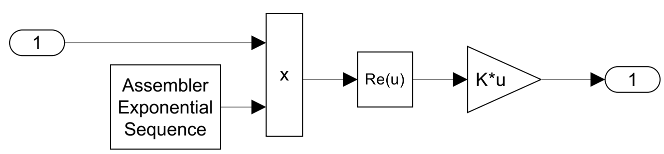

Fig. 7. Construction of the assembler.

E. Plant and Feedback

In this work, the plant block is an LTI system consisting of a transfer function block and a time delay block, as shown in Fig. 8. A disturbance input is added to the input side of the plant for this design. The reference and disturbance signals are obtained from their constant dispersion vectors through assembler blocks, as shown in Fig. 1.

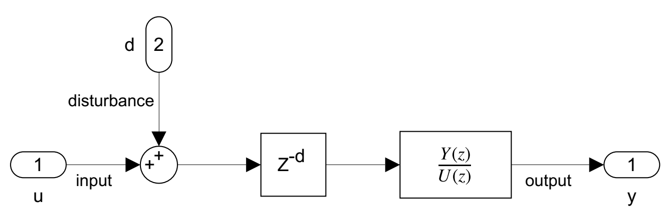

Fig. 8. A typical LTI plant model.

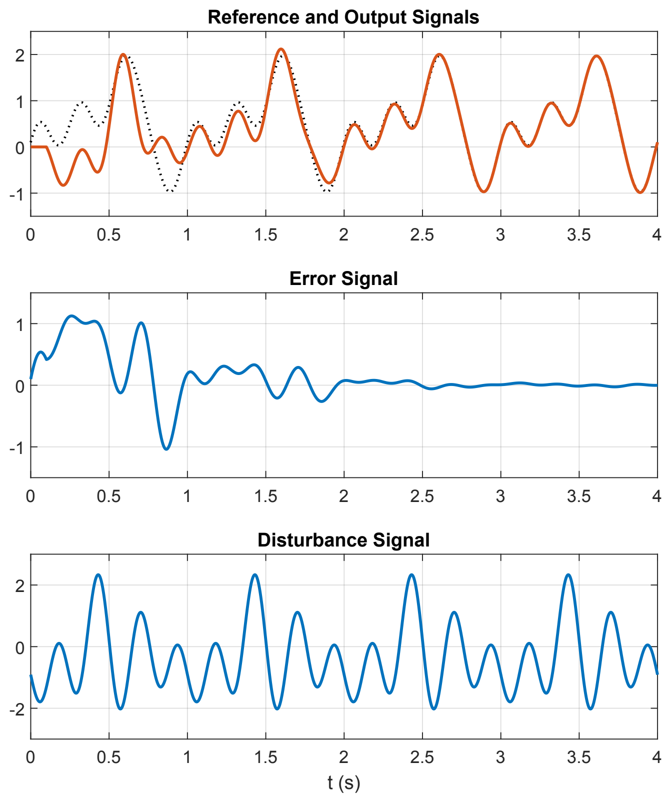

Fig. 9. The reference, system output, error, and disturbance signals.

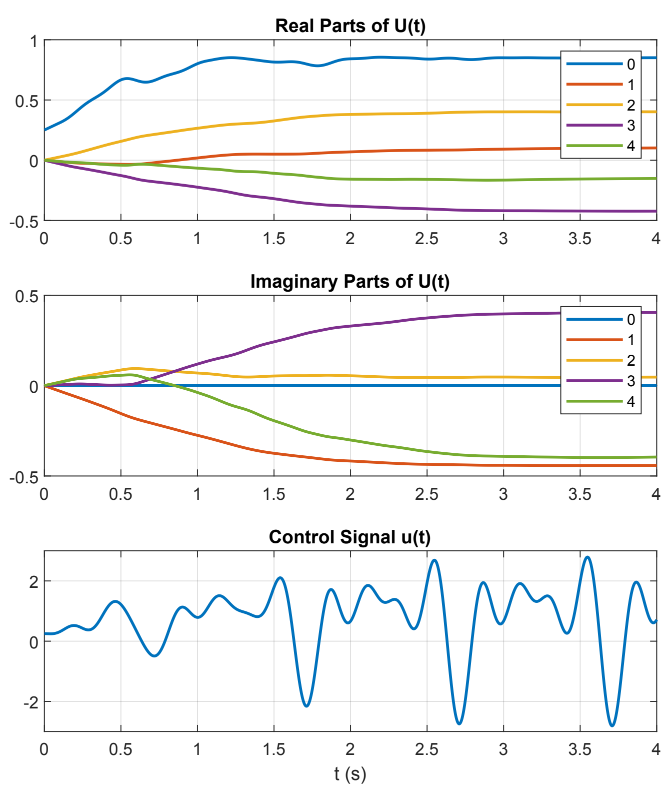

Fig. 10. Construction of the control signal from the frequency domain to the time domain.

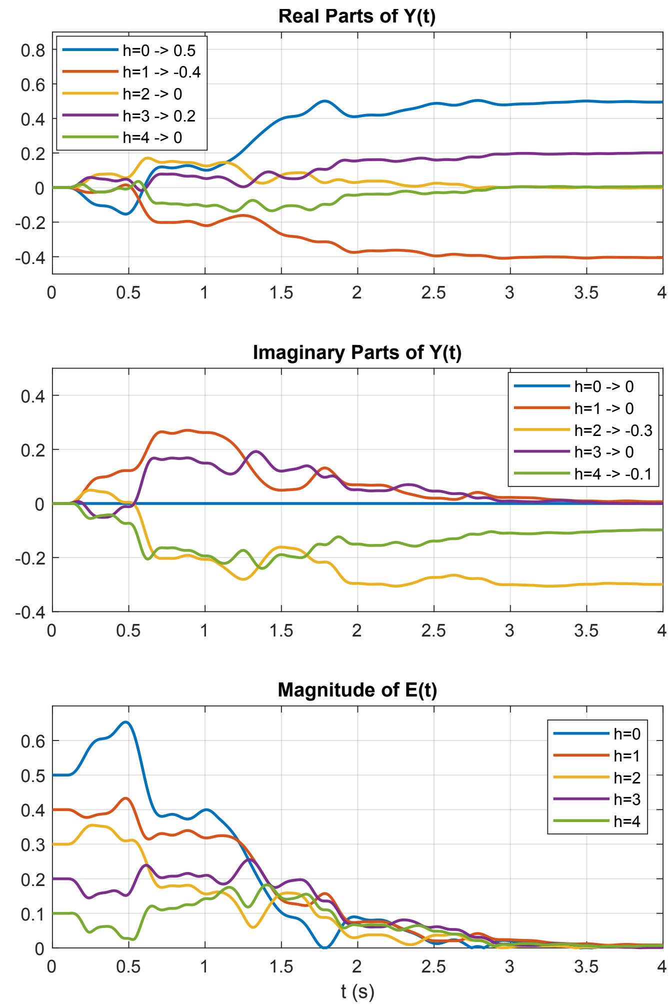

Fig. 11. Dispersions of the system output (real and imaginary parts) and the error (magnitude).

As a test system, the plant is considered to be the zero-order-hold discrete-time equivalent of

$$G(s) = \frac{10}{s+10} e^{-0.1s} \tag{11}$$

with a sampling time of 1 ms. The reference and disturbance dispersion vectors are set to be R=[0.5; -0.4; -0.3j; 0.2; -0.1j] and D=[-0.35; -0.28; 0.22j; -0.33j; 0.61j] with $H = 4$. HCA PI gains are selected as

$$K_P = \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$K_I = \begin{bmatrix} 1.2 & 0.8e^{1.2j} & 1.1e^{2.2j} & 1.5e^{3j} & 2.1e^{-2.6j} \end{bmatrix}^T.$$

The simulation results for the system output, together with the reference, error, and disturbance signals, are shown in Fig. 9. As we see, despite the disturbance and the delay, the steady state error approaches to zero in about two periods. In Fig. 10, the construction of the compensating control signal is depicted as the real and imaginary parts of its dispersion. The integrators working for each harmonic with proper complex gains gradually find the necessary harmonic control levels to make the error dispersion zero. The assembler block then

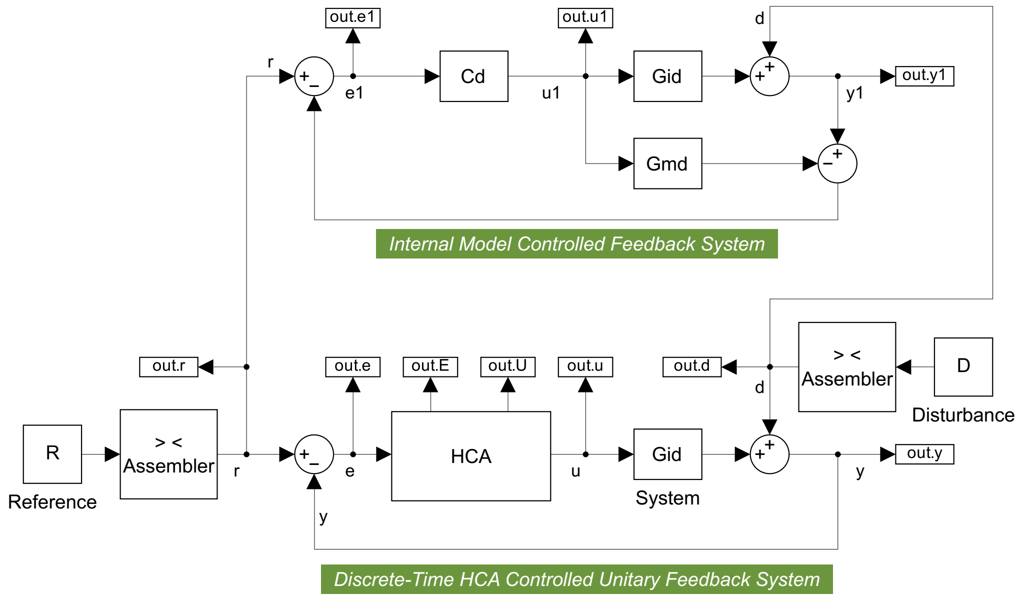

Fig. 12. Internal model control and harmonic control array structures for periodic disturbance compensation.

combines those harmonics to produce the real-time control signal to be injected into the plan input.

Fig. 11 shows the dispersion of the output and error signals. Note that the real and imaginary parts of the dispersion of the output are gradually settled at the same levels of the dispersion of the reference so that the magnitude of the dispersion of the error vanishes to zero, as shown in the figure.

IV. CONTROL OF A HOT ROLLING MILL PROCESS

A hot rolling mill process is considered in [12]. An IMC controller is systematically constructed following the design proposed in [10]. The standard IMC scheme is modified to handle a sinusoidal disturbance. The proposed IMC scheme [12] together with the HCA feedback structure is constructed as shown in Fig. 12 for the purpose of performance compression. The system transfer function is considered as

$$G_{id}(s) = \frac{K}{Ts+1}e^{-s\tau}. \quad (12)$$

G_{md} is the nominal model for the system transfer function. The nominal parameters are $T_m = 0.8s$ and $K_m = 0.5$ with the delay amount $\tau = 3.8s$. The output disturbance is considered as

$$d(t) = 0.011\sin(5.32t)(mm), \quad (13)$$

and, the reference level is chosen to be 0.05 mm.

An IMC controller with a third-order filter is proposed in [12] as

$$C_d(s) = \frac{14.28s^2 + 29.75s + 14.88}{s^3 + 1.92s^2 + 30.17s + 7.44}e^{-0.634s}. \quad (14)$$

An HCA structure is designed using the corresponding disturbance period $T = 1.18s$. The disturbance and reference are produced with assembler blocks using the constant dispersions $D = \begin{bmatrix}0 & -0.0055j\end{bmatrix}^T$ and $R = \begin{bmatrix}0.05 & 0\end{bmatrix}^T$ with $H = 1$. The proportional and integral gains are chosen as $K_p = \begin{bmatrix}0.28 & -0.4+0.45j\end{bmatrix}^T$ and $K_i = \begin{bmatrix}0.22 & -0.9+0.43j\end{bmatrix}^T$.

The simulation results are shown in Fig. 13 for the nominal model parameters and perturbed parameters. As expected, the controllers can not compensate the output for the first 3.8s due to the system delay. In Fig. 13 (a), the nominal system is simulated for both controllers. As we see, the IMC controller suppresses the oscillation disturbance faster than the HCA controller after the delay period (between 3.8 to 18 seconds). This is due to the advantage of the exact system model integrated in the IMC structure, which helps predict the system output before the added disturbance and manages the compensation process through C_d accordingly. Therefore, the oscillation errors due to the sinusoidal disturbance could be eliminated faster. On the other hand, for this setup, the HCA structure does not use or depend on any internal model of the original system, it just uses the standard PI controller for the harmonic oscillation frequency. Anyhow, we observe that both controllers have almost identical 2% settling time (~ 18 seconds).

When the system parameters are perturbed, the simulated responses are shown in Fig. 13(b,c,d). For (b), $T = 0.64s$ and $K = 0.7$, again, the controllers have similar performances, but the HCA controller compensates the sinusoidal disturbance better after the 20th second. For (c), $T = 0.5s$ and $K = 0.7$, the IMC controller does not show an acceptable performance and it is at its stability limit. For (d), $T = 0.5s$ and $K = 0.8$, on the other hand, the IMC controller makes the feedback system unstable, whereas the HCA controller still compensates for the disturbance at an acceptable performance. Therefore, it is clearly seen that better robustness is achieved with the HCA controller for this example.

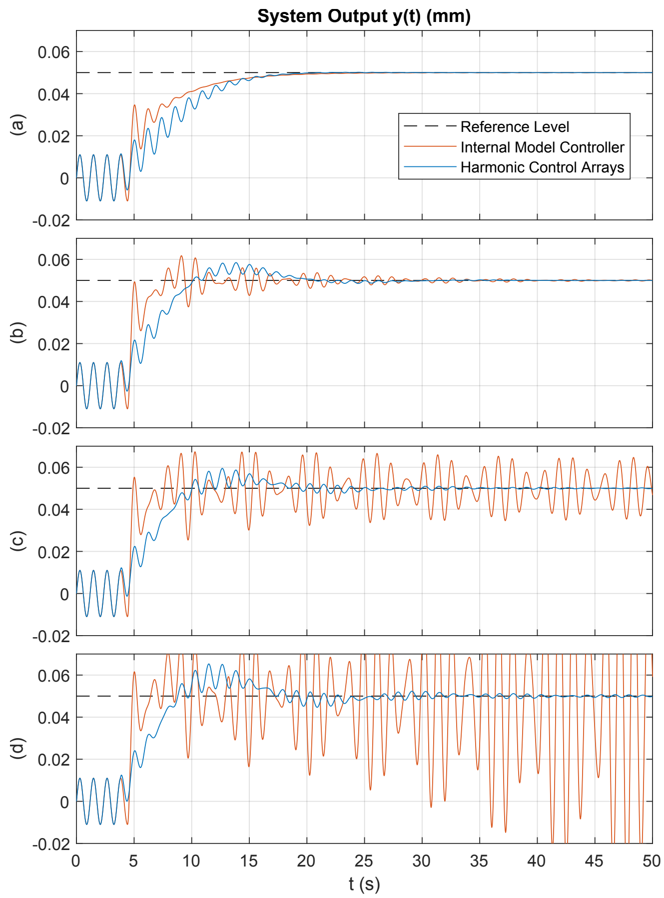

Fig. 13. Compensation of the periodic disturbance by the IMC controller and HCA controller for the nominal model parameters and perturbed parameters. (a) (nominal model) $T = T_m = 0.8s$ and $K = K_m = 0.5$, (b) $T = 0.64s$ and $K = 0.7$, (c) $T = 0.5s$ and $K = 0.7$, (d) $T = 0.5s$ and $K = 0.8$.

V. CONCLUSIONS

This paper presents the harmonic control array method with some examples. A MATLAB/Simulink modeling of HCAs is constructed using the current model library. The dispersion process continuously produces the harmonic components of a real-time signal in a vector form, the error signal in this case. The HCA internal controller block, implemented as PI controllers in this work, constructs the harmonic vector components of the control signal. Then the assembler block combines the harmonic signals into a real-time control signal. The reference and disturbance signals produced by assembler blocks from the corresponding dispersion values. The closed loop structure is completed in the simulation environment with the plant model. Various signals obtained are presented in the figures to show how the HCA constructs the necessary compensating controls. Furthermore, a hot rolling mill process example in the literature is considered. It is shown that the HCA performance is more robust than the internal model controller in this case.

Many physical systems are affected by periodic disturbances, and periodic reference tracking is necessary for some applications. Mechanical vibration suppression and noise cancellation are important control problems. Numerous industrial electronics applications, including power converters and motor drives, need to track or reject periodic signals. For these applications and more, harmonic control arrays could be a relatively simple but effective control strategy. Even for nonlinear systems, a compensating control action as a combination of many harmonic components can be automatically constructed. The PI complex gains can be obtained using various optimization algorithms available. An adaptive control strategy may also be used to reduce the transient response time. An analytic method is to be suggested to select the HCAs' complex PI control parameters so that the closed loop's stability is guaranteed.

A digital system is needed to implement the HCA method since the parameters and the signals are complex-valued. Microcontrollers and DSP devices can be readily used for this purpose. For systems with high-frequency signals or a high number of harmonics, like some power electronics or audio applications, an FPGA-based implementation may be needed. The HCA structure perfectly fits this type of implementation. Generic HCA controller devices can be produced like PID controllers in the industry for tracking or rejecting periodic signals in systems.

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