

***Modeling of Harmonic Control Arrays***  
***using MATLAB/Simulink***  
***with an Application to a Hot Rolling Mill Process***

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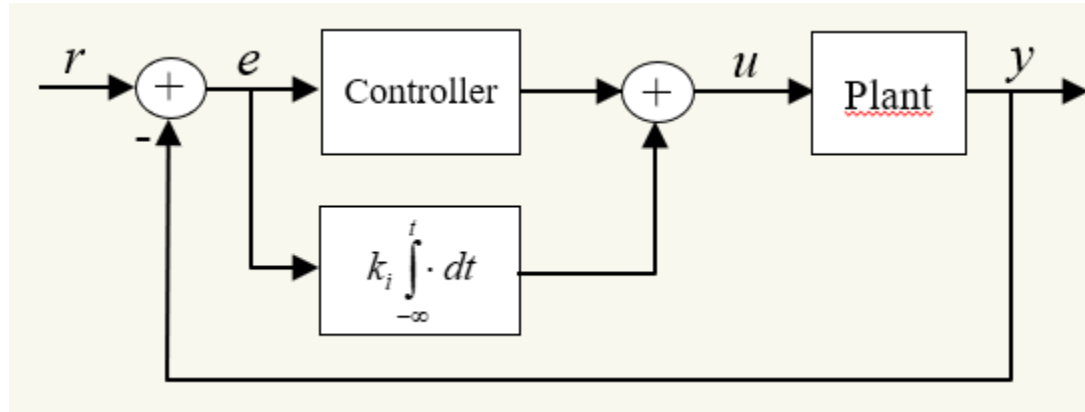
# *Outline*

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- “Harmonic Control Arrays” method is introduced
- A MATLAB/Simulink modeling of HCAs is constructed
- An example is provided for demonstration
- A hot rolling mill process is considered
- The results are summarized

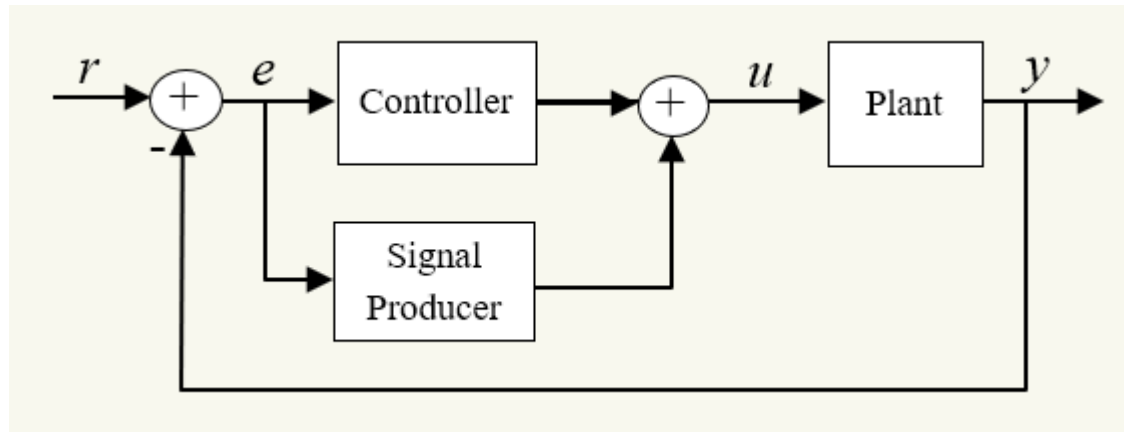
# Motivation

For constant references, the steady state error can be automatically eliminated by simply utilizing an integral term in unitary feedback systems. This is frequently and very usefully employed in many industrial control solutions.

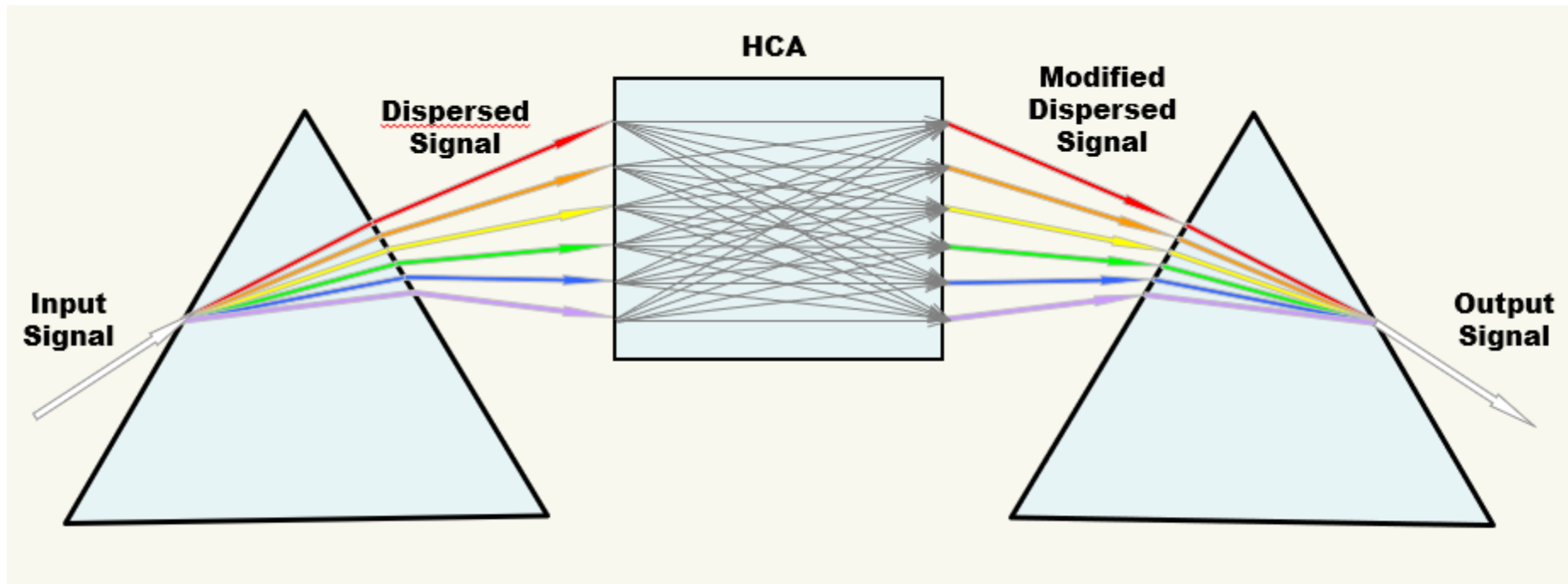


# Motivation

How can we achieve the zero-error performance for systems containing periodic references and/or periodic disturbances?



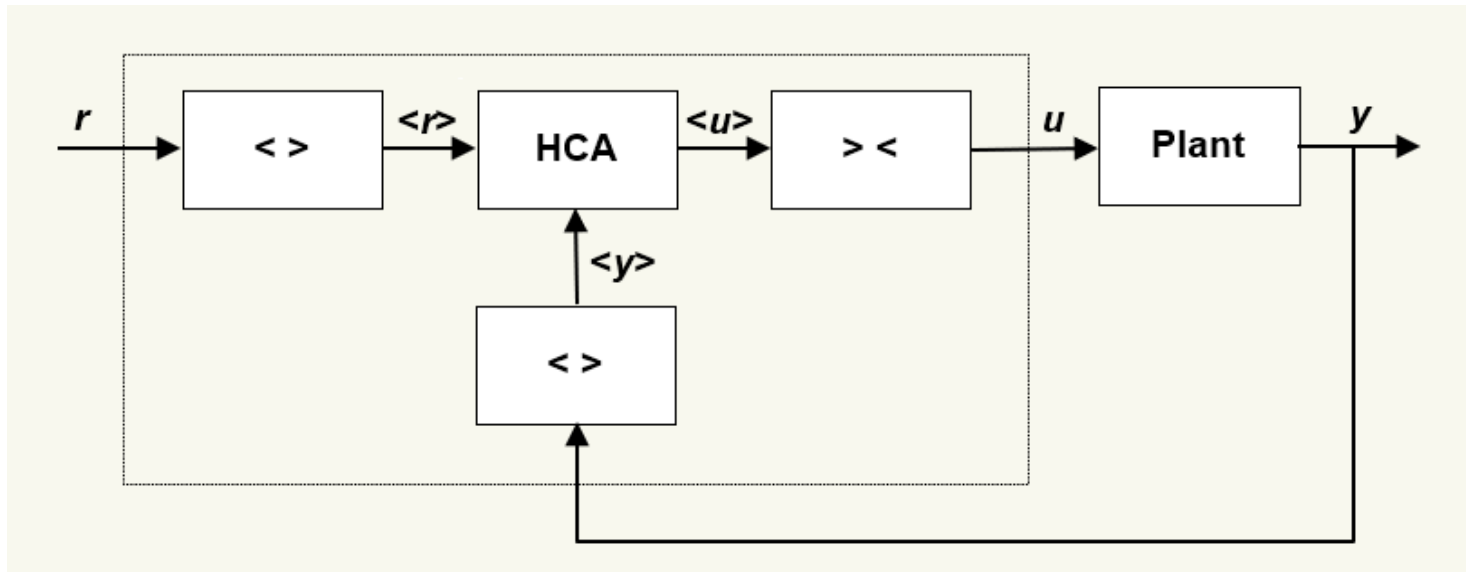
# *An Illustration of the HCA Idea*



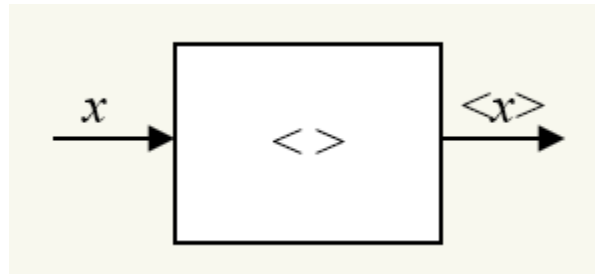
Each harmonic component is individually controlled to construct the output.

# The HCA Method

The method is based on automatically constructing the appropriate control signal by adjusting the complex levels of its harmonic components, using the harmonic components of the reference and output signals.



# Harmonic Disperser

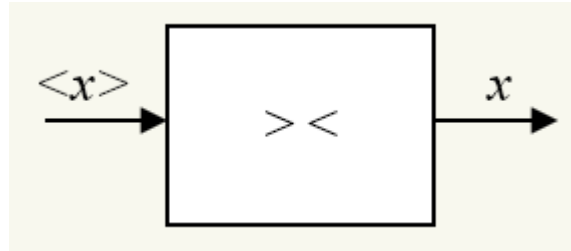


$$\langle x \rangle_h(t) = \frac{1}{T} \int_{t-T}^t x(\tau) e^{-jh\omega\tau} d\tau$$

$$\omega = 2\pi / T$$

$$\langle x \rangle = \begin{bmatrix} \langle x \rangle_0 \\ \langle x \rangle_1 \\ \vdots \\ \langle x \rangle_H \end{bmatrix}$$

# Harmonic Assembler

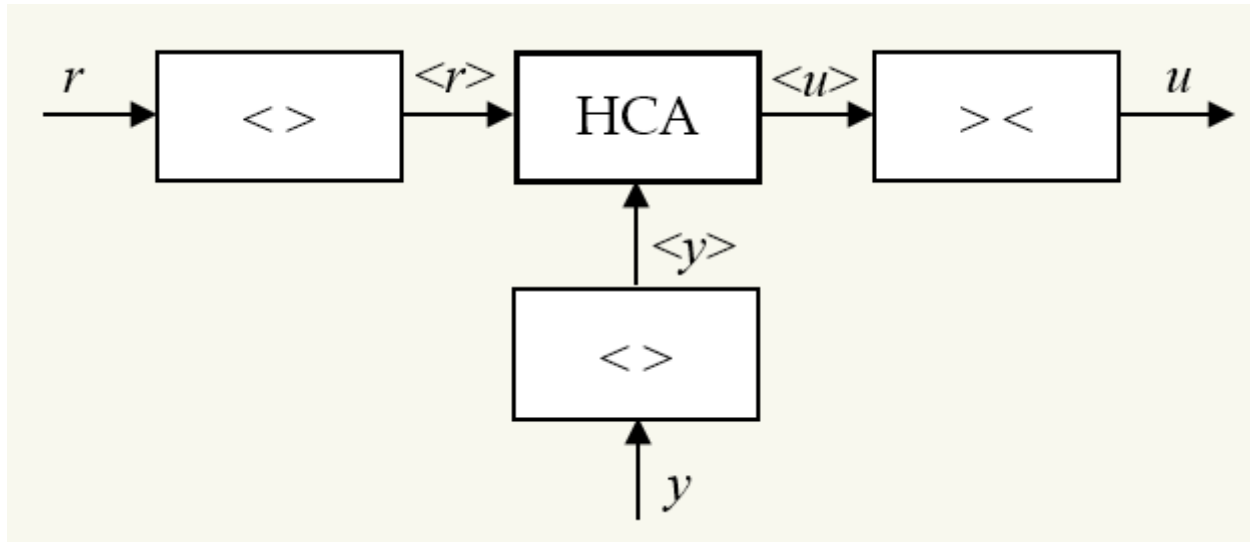


$$x(t) = \sum_{h=-H}^H \langle x \rangle_h(t) e^{jh\omega t}$$

$$\langle x \rangle_{-h} = (\langle x \rangle_h)^*$$

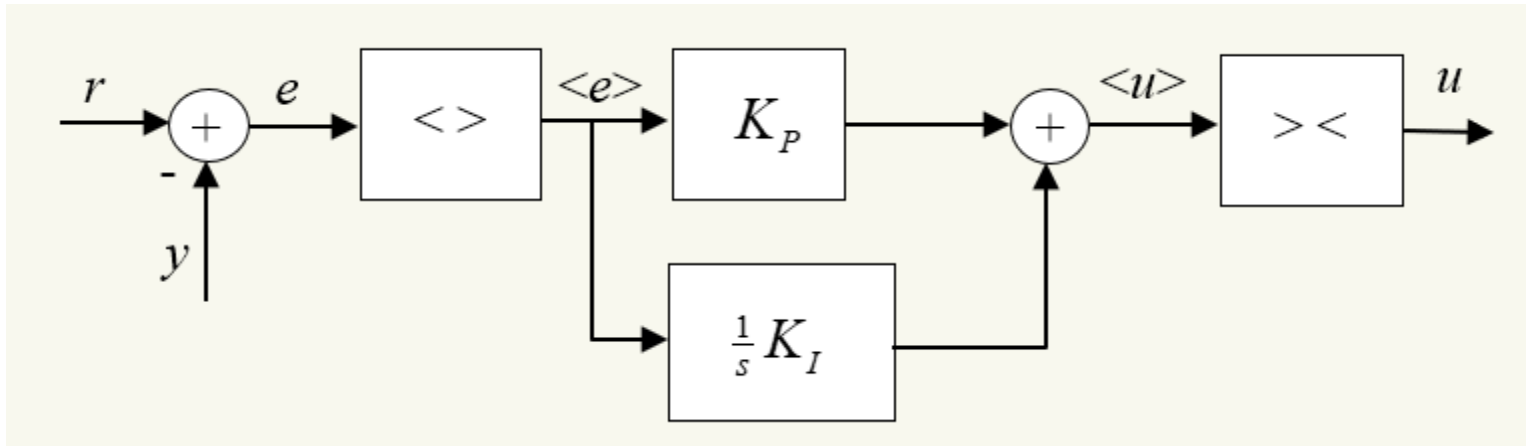


# Harmonic Control Arrays



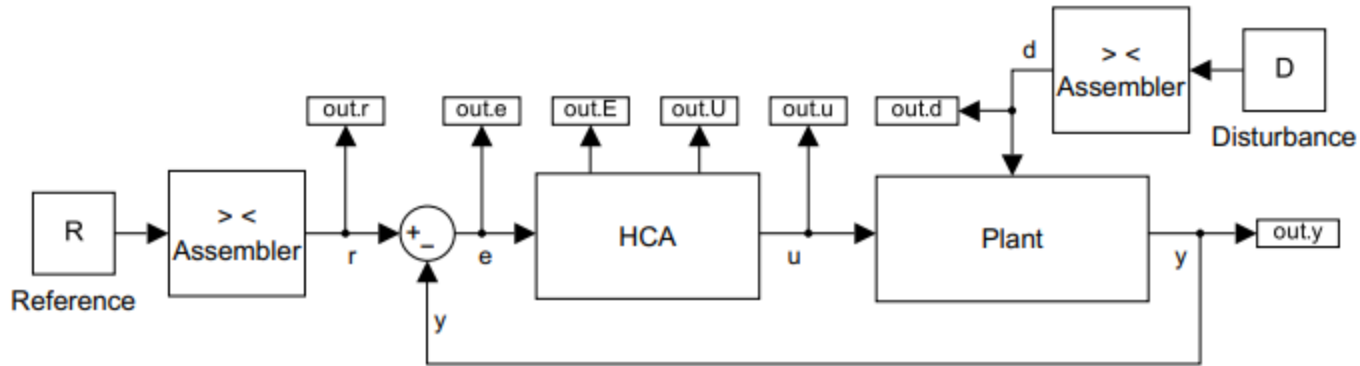
The HCA block tries to “optimally” construct  $\langle u \rangle$  using  $\langle r \rangle$  and  $\langle y \rangle$  (and their previous values).

# Harmonic PI Control Array

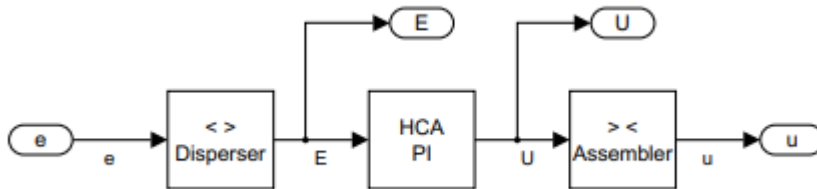


$$\langle u \rangle = K_P \langle e \rangle + K_I \int_{-\infty}^t \langle e \rangle dt$$

# Blocks in the HCA structure



Discrete-Time HCA Controlled Unitary Feedback System



**Block Parameters: HCA** ✕

PI based HCA

HCA controller with PI harmonic gains. Note that  $K_p$  and  $K_i$  complex gains are vertical vectors in this setup. Each entry belongs to the corresponding harmonic number starting from 0 to the maximum harmonic number  $H$ .

Parameters

Maximum Harmonic Number (H)  4

Number of Samples in Fundamental Period (Ns)  1000

Complex Proportional Gains ( $K_p$ )  <5x1 double>

Complex Integral Gains ( $K_i$ )  <5x1 double>

Sample Time ( $T_s$ )  0.001

Disperser Initial State (DIS)  <5x1000 double>

Initial Conditions of Integral Controllers (E0)

# Construction of a Simulink Model

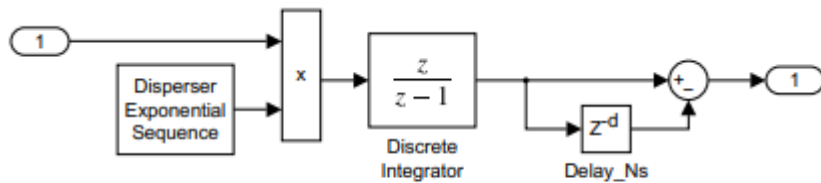


Fig. 4. Construction of the disperser.

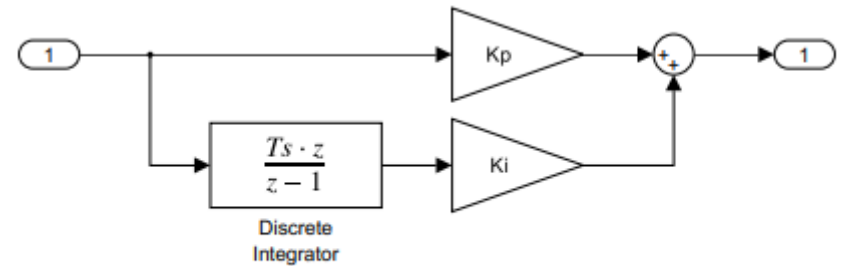


Fig. 6. Construction of the HCA PI controller.

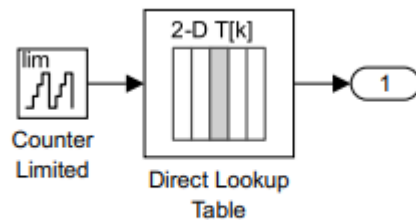


Fig. 5. Construction of the exponential sequence generator.

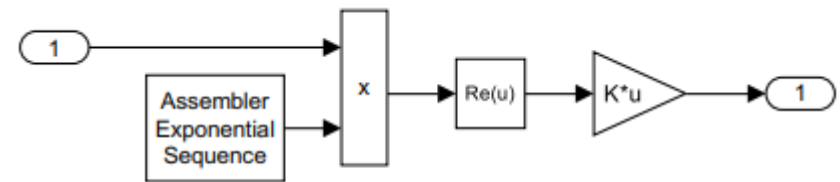


Fig. 7. Construction of the assembler.

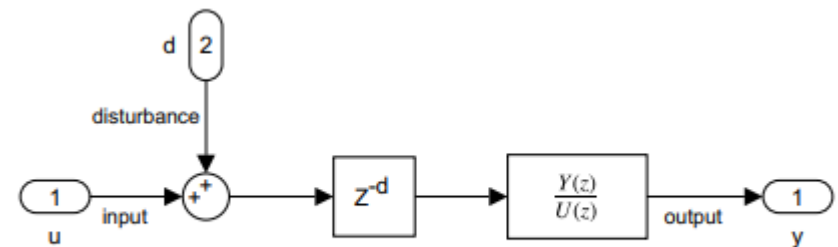


Fig. 8. A typical LTI plant model.

# A Test System

$$G(s) = \frac{10}{s + 10} e^{-0.1s}$$

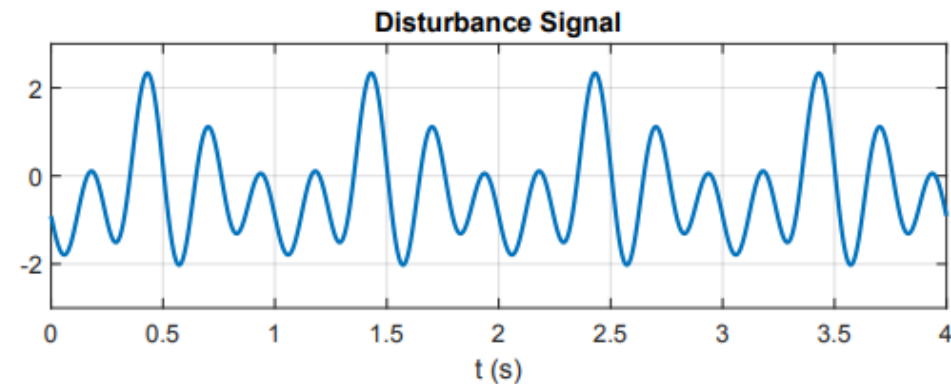
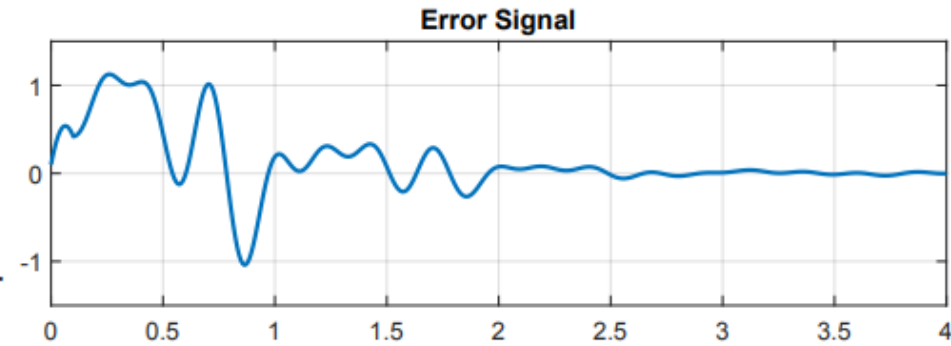
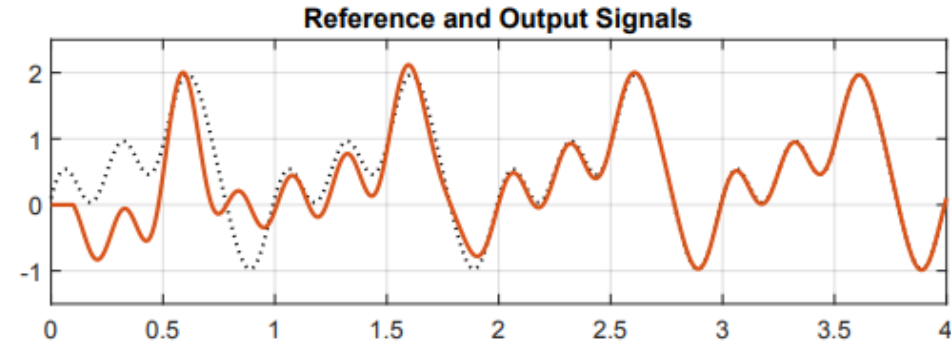
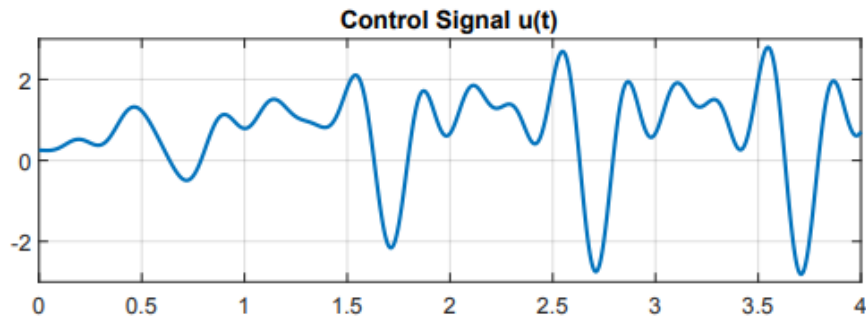
$$H = 4$$

$$R = [0.5; -0.4; -0.3j; 0.2; -0.1j]$$

$$D = [-0.35; -0.28; 0.22j; -0.33j; 0.61j]$$

$$K_P = [0.5 \ 0 \ 0 \ 0 \ 0]^T,$$

$$K_I = [1.2 \ 0.8e^{1.2j} \ 1.1e^{2.2j} \ 1.5e^{3j} \ 2.1e^{-2.6j}]^T.$$



# Control of a Hot Rolling Mill Process

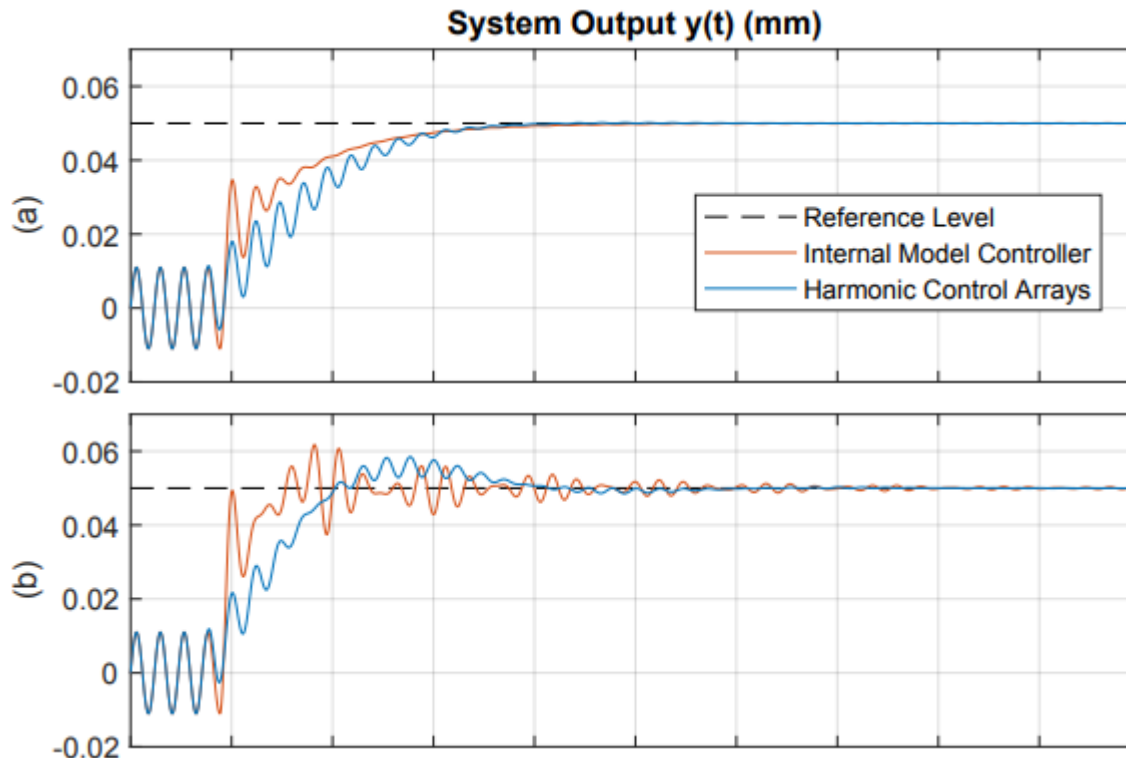
[12] Can Kutlu Yuksel, Jaroslav Busek, Silviu-Iulian Niculescu, Tomas Vyhlidal, "Internal Model Controller to Attenuate Periodic Disturbance of a First-Order Time-Delay System", 2021 European Control Conference (ECC), pp. 81-86, 2021.

$$G_{id}(s) = \frac{K}{Ts + 1} e^{-s\tau}$$

$$d(t) = 0.011 \sin(5.32t) (\text{mm})$$

An IMC controller with a third-order filter is proposed

$$C_d(s) = \frac{14.28s^2 + 29.75s + 14.88}{s^3 + 1.92s^2 + 30.17s + 7.44} e^{-0.634s}$$



$$T = T_m = 0.8s$$

$$K = K_m = 0.5$$

$$T = 0.64s$$

$$K = 0.7$$

$$T = 1.18s \quad D = [0 \quad -0.0055j]^T \quad R = [0.05 \quad 0]^T \quad H = 1$$

$$K_p = [0.28 \quad -0.4 + 0.45j]^T \quad K_i = [0.22 \quad -0.9 + 0.43j]^T$$

# Control of a Hot Rolling Mill Process

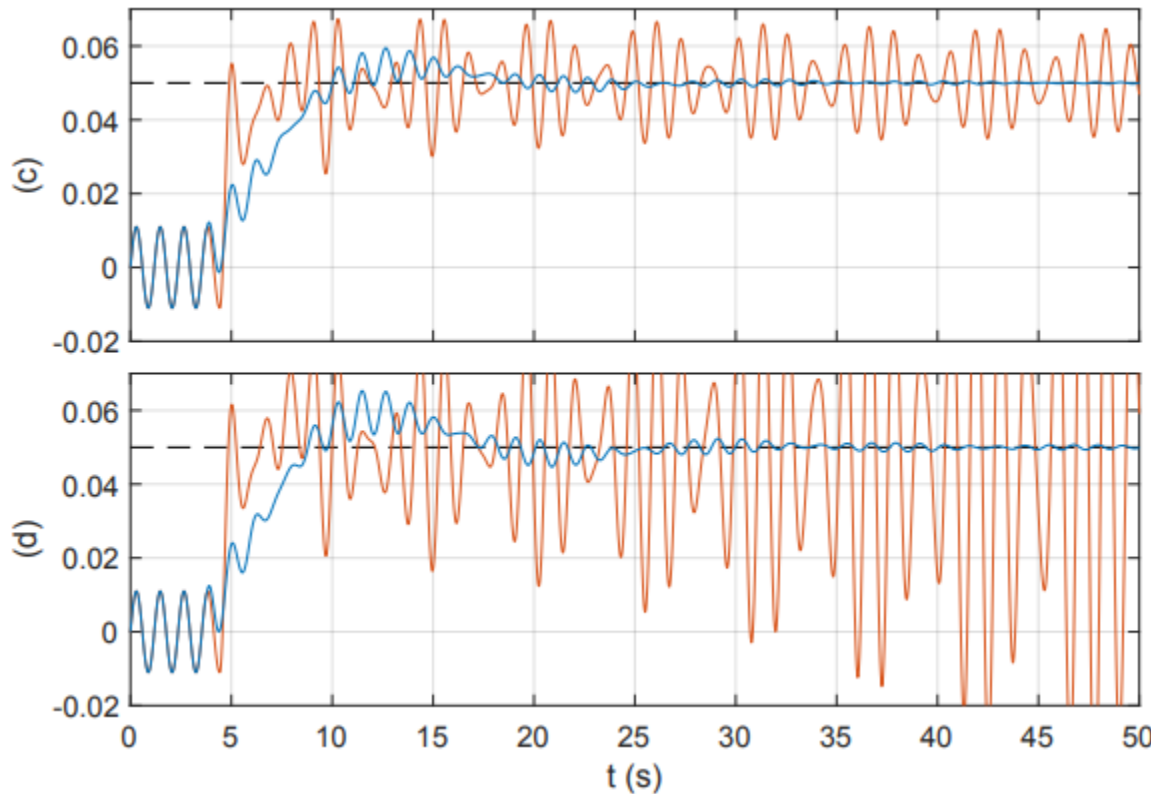
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$$C_d(s) = \frac{14.28s^2 + 29.75s + 14.88}{s^3 + 1.92s^2 + 30.17s + 7.44} e^{-0.634s}$$



T = 0.5s  
K = 0.7

T = 0.5s  
K = 0.8

# Conclusions

- Harmonic Control Array method is presented with some examples.
- A MATLAB/Simulink modeling of HCAs is constructed using the current model library.
- A hot rolling mill process example in the literature is considered.
- It is shown that the HCA performance is more robust than the internal model controller in this case.
- Harmonic control arrays could be a relatively simple but effective control strategy for periodic reference tracking, eliminating periodic disturbances, vibration suppression, noise cancellation, industrial electronics, power converters and motor drives.



# Thank You. Any Questions?

