HCA Implementation Guide

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The Harmonic Control Array (HCA) method is an effective control solution for systems with periodic references and/or disturbances. The HCA method, by employing controllers in an array structure for each dispersed harmonic components, automatically and appropriately constructs the complex levels of the harmonic components of the control input signal.

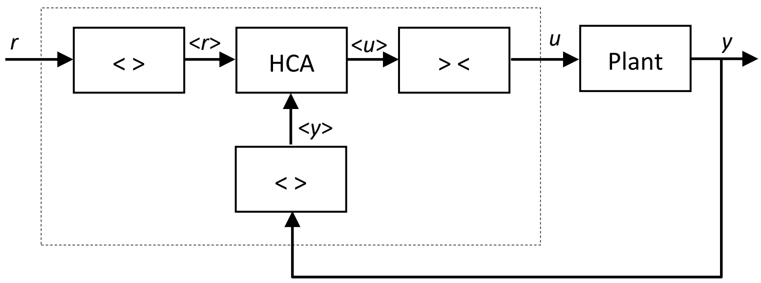

Fig. 1. Harmonic control array block diagram.

The Harmonic Disperser

A *Harmonic Disperser* produces the running harmonic components of the input signal as a function of time. *h*th *running harmonic* (or simply *h*th *harmonic*) of a signal $x(t)$ can be obtained using a running Fourier series integral as

$$< x >_h (t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) e^{-jh\omega\tau} d\tau \qquad (1)$$

where h is an integer number, T represents the fundamental period and it is chosen as a fixed value by the designer, and $\omega = 2\pi/T$ is the angular frequency. $x(t)$ is assumed to be real valued.

$<x>_h$ is closely related to Fourier series coefficients. In fact, if the signal $x(t)$ is actually periodic with T, $<x>_h$ will be a complex constant value in time, and equal to the *h*th Fourier series coefficient. Considering such harmonics from 0 to H, *harmonic dispersion* of $x(t)$ is defined as

$$< x > = \begin{bmatrix} < x >_0 \\ < x >_1 \\ \vdots \\ < x >_H \end{bmatrix} \qquad (2)$$

If $<x>$ is constant in time (and $x(t)$ does not contain any harmonics higher than H), $x(t)$ is periodic with T.

The Harmonic Assembler

A *Harmonic Assembler* recombines a signal from its running harmonic components. Borrowing from Fourier series synthesis equation, the signal is produced using

$$x(t) = \sum_{h=-H}^{H} < x >_h (t) e^{jh\omega t} . \qquad (3)$$

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Here, the negative harmonic components are also needed to construct $x(t)$. Since x is assumed to be real valued, $<x>_{-h}$ is equal to the conjugate of $<x>_h$ (that is $<x>_{-h}=<x>_h^*$). Therefore $<x>$, as containing the harmonics 0 to H, is sufficient to produce x. From Equation 3, an equivalent representation involving only nonnegative harmonics is

$$x(t) = < x >_0 (t) + 2Re\left\{ \sum_{h=1}^{H} < x >_h (t) e^{jh\omega t} \right\}. \quad (4)$$

Harmonic PI control array block diagram.

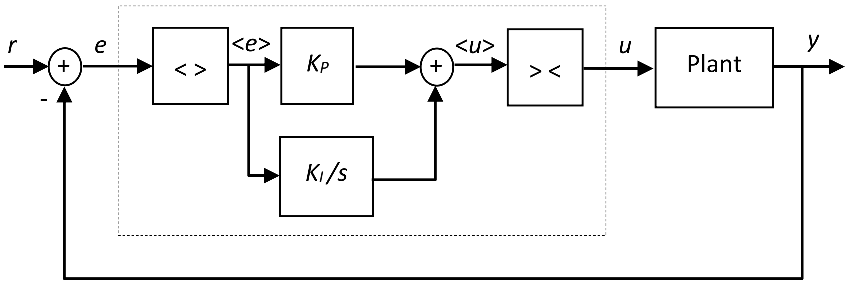

Fig. 2. Harmonic PI control array block diagram.

Assuming the HCA block is constructed using PI controllers, the corresponding control diagram is as shown in Fig. 2, and, the harmonic dispersion of the control signal is calculated as

$$< u > = K_P < e > + K_I \int_{-\infty}^{t} < e > dt \quad (5)$$

where $e = r - y$ is the error signal. Note that even if the system is SISO, the dispersion variables will be vectors of dimension $(H+1)\times 1$. Therefore, K_P and K_I are the proportional and integral gain matrices with proper dimensions, and possibly having complex valued entries. In SISO case, these are square matrices with dimension $(H+1)\times(H+1)$. If each harmonic has a feedback only to the same harmonic (which may be suitable for sufficiently linear plants), then these gain matrices are diagonal. Instead of only one PI controller as in classical control, an array of PI controllers are, therefore, acting in parallel on each harmonic to compose the final control signal.

Discrete Time Implementation of Harmonic Disperser

Equivalent to continuous counterparts, discrete time implementation formulas for the Harmonic PI control array can be given as

$$\begin{aligned} < x >_h [n] &= \frac{1}{N} \sum_{k=n-N+1}^{n} x[k]\, e^{-jh\omega k T_s} \\ &= \frac{1}{N} \sum_{k=n-N+1}^{n} x[k]\, e^{-j2\pi h k/N} \qquad (6) \end{aligned}$$

where T_s is the sampling period, $x[k] = x(kT_s)$, $N = T/T_s$, and $<x>_h[n]$ approximately represents $<x>_h(nT_s)$. Note that the exponential term in (6) is a periodic function in time, that is

$$e^{-j2\pi hk/N} = e^{-j2\pi h(k+N)/N} \quad (7)$$

for each $k \epsilon \mathbb{Z}$, because of periodic nature of the equation (7), it can be calculated for only one period, for a total of N cases. After these values are calculated for $k = 0, 1, \ldots, N - 1$, instead of recalculating each time, these values can be stored to the memory, and be recalled whenever they are to be used. Another important calculating time saving can be achieved by considering that the sum in (6) is carried out for a limited period, and many common terms are present in addition. To avoid making extra identical calculation, it can alternatively be determined as

$$< x >_h [n] = < x >_h [n-1] + \frac{1}{N}(x[n] - x[n-N])e^{-\frac{j2\pi hk}{N}} \quad (8)$$

Algoritm for the Harmonic PI Control Array

System shown in Fig. 2, can be implemented in discrete time as follows. First, choose the fundamental period T (for a 50 Hz signal, T=0.02). Choose the sampling period T_s, and let $N=T/T_s$ which represents the number of sampling points in a fundamental period. Choose H as the highest harmonic number that the system need to be controlled up to. Choose Kp and Ki gain matrices (in diagonal form) such that their values can be complex except the fundamental components.

For each n value:

$e[n] = r[n] - y[n]$

For each h value in the range of 0 to H:

$$< e >_h [n] = < e >_h [n-1] + \frac{1}{N}(e[n] - e[n-N])\, e^{-\frac{j2\pi hk}{N}}$$

Calculate the error integral in vector form as

$E[n] = E[n - 1] + T_s<e>[n]$

Calculate the control signal in frequency domain in vector form as

$<u>[n] = K_p<e>[n] + K_i E[n]$

The control signal in time domain can be finally calculated as

$$u[n] = < u >_0 [n] + 2Re\sum_{h=1}^{H} < u >_h [n]e^{\frac{j2\pi hn}{N}}$$

References

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