Harmonic Control Arrays Method With a Real Time Application to Periodic Position Control

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***Abstract*—A novel method is presented for systems with periodic references and/or disturbances by employing controllers in an array structure for each dispersed harmonic components. The method is based on automatically and appropriately setting the complex levels of the harmonic components of the control signal. Both computer simulation and real time experimental results are presented to illustrate the usefulness and effectiveness of the proposed method.**

***Index Terms*—Fourier series, harmonic assembler, harmonic disperser, harmonic distortion compensation, periodic control, periodic tracking.**

I. Introduction

THIS paper provides a formal introduction of the harmonic control array (HCA) method for controlling systems involving periodic reference and/or disturbance signals, including a practical application. Preliminary ideas of this novel method were presented in [1] and [2]. The HCA structure automatically constructs and injects the compensating periodic control signal into the plant input. Without necessarily employing the internal model principle, the HCA method can be adapted and integrated with various existing control techniques, enabling a perfect periodic reference tracking and disturbance rejection.

The HCA idea may be illustrated conceptually using a light-prism mechanism as shown in Fig. 1. First, the incoming light, representing the input signal, is dispersed to its frequency components as different colors representing the harmonics. In the implementation of the method, this is achieved by a *harmonic disperser* producing the complex time-variant Fourier series coefficients. Then, each frequency component is treated separately, mixed together, amplified or reproduced in an array structure in the HCA block which can be implemented as a linear or nonlinear dynamical system carrying out the required transformation or compensation. Finally, the modified frequency components assembled together again to obtain the composed output signal using a *harmonic assembler* performing the Fourier series synthesis formula. Considering the feedback systems terminology, the input signal here can represent the error signal, and the output is the compensating control signal applied to the controlled plant. The further theoretical descriptions of these structures are described in Section II.

To achieve the zero error for periodic references or disturbances, various methods are developed. One of the most well known among these is the internal model principle (IMP) [10] which states that the controlled output can track a class of reference inputs without a steady state error if the generating model of the reference signal is included in the stable closed-loop system. To compensate for sinusoidal disturbances or track sinusoidal reference signals, a resonant band pass filter having a couple of complementary pure imaginary poles at the matching frequency is added to the controller [11]–[17].

Developing control methods for tracking or rejection of periodic signals in power electronics applications such as ac/dc converters, uninterruptible power systems, active filters, and high performance motor drives is especially important. The methods of multisynchronous PI controller or multiple reference frame controller [18], rotating reference frame controller [19], synchronous regulator [20], multiple rotating integrator controller [21], and synchronous frame harmonic controller [22] are based on the frequency displacement process where the error signal is premultiplied and postmultiplied by a frame transformation. These algorithms are also applied on digital signal processors for control of UPS systems [23], photovoltaic inverters [24], and ac/dc converters [25], where digital signal processing (DSP) detects the harmonic distortion signal within the output voltage waveform and determines the amplitude of the real and imaginary parts of the harmonic components.

In these synchronous frame harmonic control techniques, first by multiplying with sinusoidal signals at a rotating frequency, the system variables are transformed to rotating frame quantities, that is like in demodulation process, the signal around a given harmonic frequency is shifted to the base band in the frequency domain. At this stage, the considered harmonic frequency of a disturbance becomes a dc quantity. Then, the signal in the synchronous frame is compensated by a proportional integral or other control technique to guarantee zero steady-state error for the considered harmonic frequency. Finally, using a modulation process, the compensator outputs are converted back to the stationary reference frame [22]. The method presented in this paper, although similar to this idea, has fundamental differences. To represent the harmonic values correctly we use the complex Fourier series integral [3] as in the dynamical phasor representation [4]–[6]. Given a running signal, this Fourier-based computation naturally gives the best harmonic representation in the sense of available data in the

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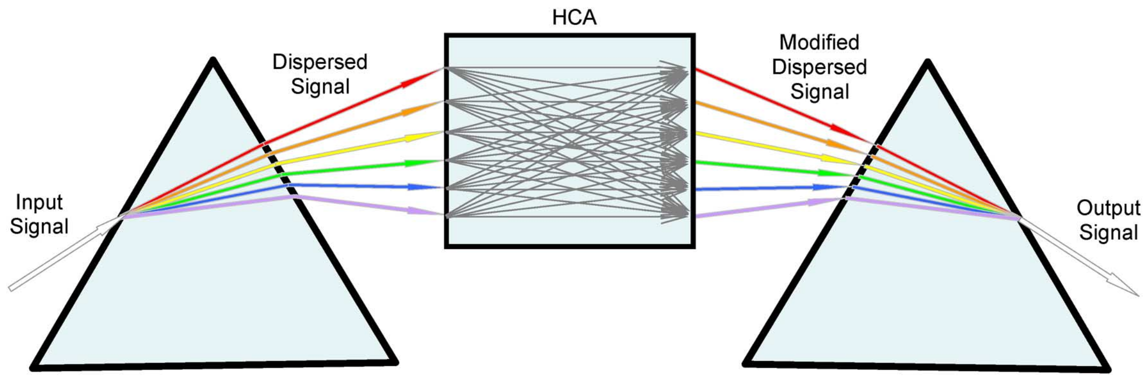

Fig. 1. Illustration of the HCA idea.

last period time of the signal. At any given time, using only the last full period is important to immediately reflect the harmonic changes in the signal. However, the band pass filtering type harmonic detection methods, although decaying in an exponential way, will carry the effect of the older data, therefore results in a slower response than Fourier series integral. Also, the Fourier series integral naturally and totally eliminates the effects of other harmonics which are generally present in the given signal. In the above methods of synchronous frame control, on the other hand, since the frequency shifting or demodulation process actually keeps all the frequencies but only changes their positions, the effects of all other harmonic signals will appear in the synchronous frame, and, may disturb the control performance or may lead to instability. Furthermore, our control technique is in more systematic and simpler structure that may be applicable to numerous control systems in a modular way.

By applying the frequency shifting properties of the Laplace transform, it is shown in [26] and [28] that, the synchronous frame controller is actually equivalent to the well known linear second order resonant compensator having an infinite gain at the resonant frequency and therefore achieving zero steady state error as used in the internal model control. Therefore, the complexity regarding to the frame transformations involved in the synchronous frame description can be avoided, and instead, a set of band pass filters compensating for selected harmonics may be utilized as an equivalent representation in the stationary reference frame. This technique is referred in the power electronics literature as multiresonant controller or Stationary frame resonant regulator [26], [27], stationary frame controller, or bank of resonant filters [28]–[30], sinusoidal internal model compensator [31], stationary frame generalized integrator [32], and single synchronous frame hybrid controller [33]. As an alternative to the use of the bank of resonant filters for the compensation of active power filters, selected harmonics are obtained by a discrete Fourier transform (DFT) or discrete cosine transform (DCT)-based running finite impulse response filters in the time domain, and compensated by a closed-loop repetitive-based control scheme [34]. It is shown in [35] and [36] that a bank of resonant filters with a particular structure used as compensators for certain harmonics is equivalent to a repetitive scheme using a simple feedback array with a delay line.

Another control method for systems involving periodic exogenous signals is the repetitive control. This method employs an artificial delay line in a self loop to produce the necessary periodic control signal, see [37] and the references therein. The repetitive control is generally applied to reject periodic disturbances and to track periodic reference signals with a known period. A number of repetitive control schemes have been developed and applied to various applications such as harmonic compensation, industrial robots, disc drives, numerical control machines [38]–[47]. The disadvantage of repetitive control, on the other hand, is that the systems without the exact models may easily be led to an unpredictable or unstable behavior because of the artificially inserted delay. This limits the usefulness of repetitive control in industrial applications. The HCA method, on the other hand, requiring no delay, and adapting a simple proportional-integral-differential (PID) strategy in the harmonic control array structure for instance, can be effectively used in industrial applications.

Mentioned compensating techniques above are based on the well-known internal modal principle, and, including the synchronous frame controller as shown to be equivalent to a set of resonant regulators, have a linear time invariant (LTI) system structure. In this regard, these techniques together with the other modern LTI techniques have a well known structure and achievement in applications at a certain level. However the HCA, to obtain the harmonic components correctly and as fast as possible, and, to eliminate the effects of all other harmonics, directly uses the running Fourier series integral in its complex nature. The block producing these dynamic phasors, called the harmonic disperser, together with its complementary block, called the harmonic assembler, becomes a time variant system. Therefore inherently, the HCA compensator is not a time invariant system as usual, and according to the employed strategy, it can be either linear or nonlinear.

The PID control is most frequently and very usefully employed for many industrial control problems. Its simple structure and effectiveness make it very favorable for engineers. It is well known that 90% or more of controllers in industry are PID. One of the most important features of PID control is achieving the zero steady-state error even if there are stationary disturbances or parameter perturbations which are abundant in real applications. Although the error entering into the integrator is zero, the integrator can produce the required compensating dc control signal. To achieve the zero error for systems involving periodic reference inputs and/or disturbances on the other hand,

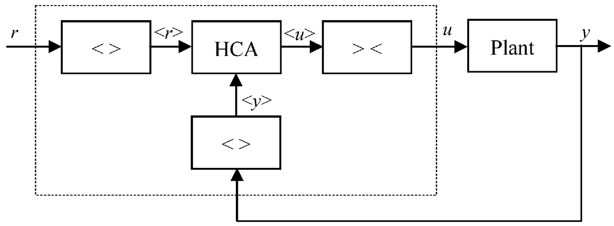

Fig. 2. HCA block diagram.

the PID control is not directly applicable, because instead of a constant compensating dc value, a special compensating periodic control signal is to be automatically constructed and injected into the plant input.

For feedback systems involving periodic references or disturbances, the zero error can be achieved by automatically composing the control signal at the right amounts of complex levels for each of the harmonic frequencies. For this purpose, an array of PI controllers may be easily employed in the harmonic control array structure. Therefore, the HCA method may allow the PID control to be effectively applied to the systems involving periodic signals and achieving the desired zero error. The other control methods may also be utilized in implementing a harmonic control array, or an HCA can be used in parallel with the other methods. The HCA method looks promising for many industrial applications on periodic tracking and periodic harmonic distortion compensation like in power electronic applications, and in mechanical vibration or noise cancellation type problems.

The following exposition is planned for this paper. First, the HCA method is described in Section II. Then, the method is simulated on a typical system model with various cases in Section III. Experimental results on a linear periodic position control application are provided in Section IV. Finally, the conclusion is provided in Section V.

II. Description of HCAs

Considering a typical system to be feedback controlled, the HCA structure can be briefly shown as in Fig. 2. Here, r is the reference input, u is the control signal, and, y is the system output. The other blocks and signals in the diagram are explained in the following subsections.

A. Harmonic Disperser

A *Harmonic Disperser* [see Fig. 3(a)] produces the running harmonic components of the input signal as a function of time. hth *running harmonic* (or simply hth *harmonic*) of a signal $x(t)$ can be obtained using a running Fourier series integral as

$$\langle x\rangle_h(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)e^{-jh\omega\tau}d\tau \qquad (1)$$

where h is an integer number, T represents the fundamental period and it is chosen as a fixed value by the designer, and $\omega = 2\pi/T$ is the angular frequency. $x(t)$ is assumed to be real valued. In the future versions of this method, T can be considered as a system variable adaptively changed for an optimum control performance.

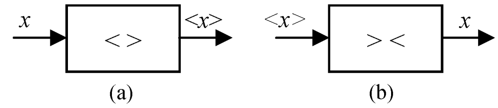

Fig. 3. Harmonic operations: (a) a harmonic disperser and (b) a harmonic assembler.

$\langle x\rangle_h(t)$ is generally a complex value representing the time-variant Fourier coefficient, and, also called an $h\omega$-component lagged running average [3] or a dynamical phasor [4]. Using the dynamical phasor representation, modeling of various electric machines and power system components is described and some properties of this operator are provided in [5], [6], and the references therein. Choosing a suitable sampling period by the designer, the Fourier integral (1) can be calculated in the discrete time by a DSP or a microcontroller. Goertzel algorithm [7]–[9] provides a very efficient calculation method, and saves computation time as well as storage memory. Like in almost all systems, and especially as in causal filters, the Fourier integral (1) has also an inherent delay. However, (1) uses the very last period to correctly assess the harmonic values and to immediately reflect the harmonic changes in the signal.

$\langle x\rangle_h$ is closely related to Fourier series coefficients. In fact, if the signal $x(t)$ is actually periodic with T, $\langle x\rangle_h$ will be a complex constant value in time, and equal to the hth Fourier series coefficient. x can also be a vector, in that case, $\langle x\rangle_h$ is also a vector with the same dimension. Considering such harmonics from 0 to H, *harmonic dispersion* of $x(t)$ can be defined as

$$\langle x\rangle = \begin{bmatrix} \langle x\rangle_0 \\ \langle x\rangle_1 \\ \vdots \\ \langle x\rangle_H \end{bmatrix}. \qquad (2)$$

If x is an n-dimensional vector, $\langle x\rangle$ will be an nH-dimensional vector. If $\langle x\rangle$ is constant in time (and $x(t)$ does not contain any harmonics higher than H), $x(t)$ is periodic with T.

Here, H is a design parameter representing the maximum harmonic number considered. H can be decided according to the process needs and the calculation power by the designer. If more harmonics are needed to be controlled in the system then H can be increased, but in this case naturally, the computational load will also increase and a tradeoff may be needed to manage all the calculations in one sampling period. However, with the increasing computational power of DSP devices and the efficient algorithms developed, the feasible implementation of these kinds of operations with high harmonic numbers are becoming possible for even fast sampling frequencies.

B. Harmonic Assembler

A *Harmonic Assembler* [see Fig. 3(b)] recombines a signal from its running harmonic components. Borrowing from Fourier series synthesis equation, the signal is produced using

$$x(t) = \sum_{h=-H}^{H} \langle x\rangle_h(t)e^{jh\omega t}. \qquad (3)$$

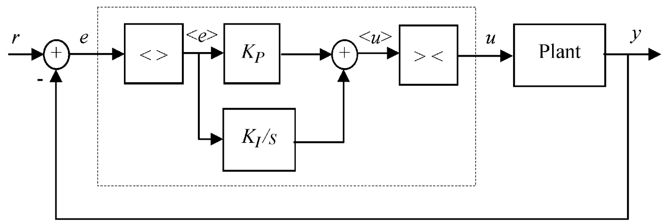

Fig. 4. Harmonic PI control array block diagram.

Here, the negative harmonic components are also needed to construct $x(t)$. Since x is assumed to be real valued, $\langle x\rangle_{-h}$ is equal to the conjugate of $\langle x\rangle_h$ (that is $\langle x\rangle_{-h} = \langle x\rangle_h^*$). Therefore $\langle x\rangle$, as containing the harmonics 0 to H, is sufficient to produce x. From (3), an equivalent representation involving only nonnegative harmonics is

$$x(t) = \langle x\rangle_0(t) + 2Re\left\{\sum_{h=1}^{H}\langle x\rangle_h(t)e^{jh\omega t}\right\}. \tag{4}$$

C. HCA Block

Using $\langle r\rangle$ and $\langle y\rangle$, and possibly their previous values in time, the HCA block in Fig. 2 produces harmonic dispersion of the control signal $\langle u\rangle$ in an optimum way as much as possible. This can be achieved employing various control methods like linear, fuzzy, sliding mode, adaptive, robust, depending on the designer's choice. Therefore, the control action works by firstly obtaining the harmonic dispersions of the reference and the system output, then, according to these complex values, the HCA decides on $\langle u\rangle$, and finally the real control input to be applied is composed by a harmonic assembler. There are some complex valued calculations involved in this kind of control; therefore an analog circuit may not be suitable to realize it. Instead, the algorithm can be easily and flexibly implemented on a microcontroller or a digital signal processor. With the higher calculation power and advanced speeds of the digital devices, today it is possible to implement such control algorithms even for relatively fast applications. In the present paper, the HCA block is assumed to be constructed using PI controllers for simplicity. For this case, the corresponding control diagram is as shown in Fig. 4, and, the harmonic dispersion of the control signal is calculated as

$$\langle u\rangle = K_P\langle e\rangle + K_I\int_{-\infty}^{t}\langle e\rangle dt \tag{5}$$

where $e = r - y$ is the error signal. Note that even if the system is single-input-single-output (SISO), the dispersion variables will be vectors of dimension $(H+1)\times 1$. Therefore, K_P and K_I are the proportional and integral gain matrices with proper dimensions, and possibly having complex valued entries. In SISO case, these are square matrices with dimension $(H+1)\times(H+1)$. If each harmonic has a feedback only to the same harmonic (which may be suitable for sufficiently linear plants), then these gain matrices are diagonal. For nonlinear plants, on the other hand, since the lower harmonics in the plant input may affect the higher harmonics, the off-diagonal entries may be needed for compensation or better control performance. If needed, an *anti-windup* protection algorithm can be used for the integral term in (5). However here, since the values to be integrated are complex numbers in general, the saturation should be activated when the *magnitude* of the integrated complex values are reached to the maximum value foreseen at the design stage.

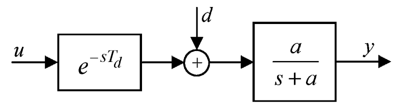

Fig. 5. Typical system model.

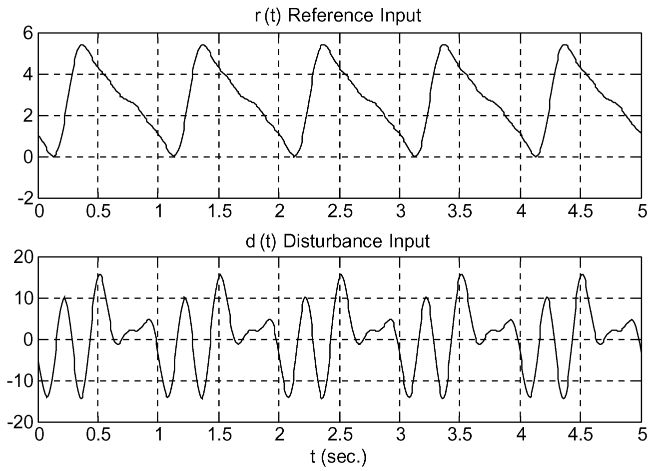

Fig. 6. Reference and disturbance inputs.

Instead of only one PI controller as in classical control, an array of PI controllers are, therefore, acting in parallel on each harmonic to compose the final control signal. This, in general, suggests the name "HCA" to describe this kind of control structure.

III. Numerical Simulation Examples

To investigate how the HCA method performs, let us consider a typical system model provided in Fig. 5. The system is linear having a pole at $-a$ and an input delay of T_d together with a disturbance input. a equals to 1 unless stated otherwise.

The reference that the system output must follow and a powerful sample disturbance signal used are shown in Fig. 6. For these signals, H is chosen to be 4, and the period T is 1 s. Corresponding constant harmonic dispersions are as follows:

$$\langle r\rangle = \begin{bmatrix} 2.7 \\ -1 \\ 0.5\text{j} \\ 0.2 \\ -0.1\text{j} \end{bmatrix}, \qquad \langle d\rangle = \begin{bmatrix} 0.35 \\ -1.6+1.6j \\ 0.25-0.3j \\ -2.95+3.05j \\ 2.5+1.25j \end{bmatrix}. \tag{6}$$

The system behaviors for different cases are investigated in the following sections.

A. Behavior of the System Without Delay

First, consider the system without delay ($T_d = 0$). Apply the harmonic PI control array method shown in Fig. 4 with PI gains selected as

$$K_P = \text{diag}(2.5, 10, 15, 20, 20) \quad K_I = \text{diag}(0.9, 4, 10, 10, 10). \tag{7}$$

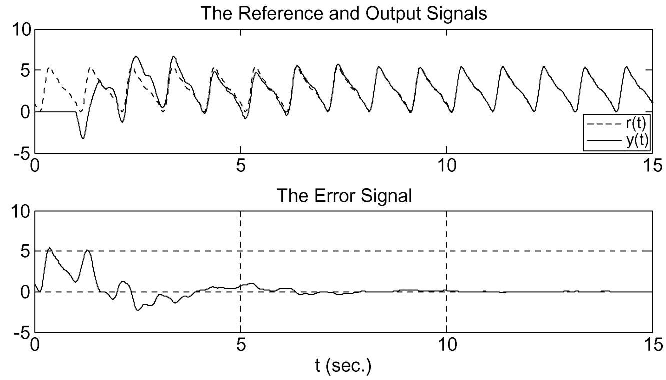

Fig. 7. Output and error in controlling with the HCA method for the system without delay.

The system response is given in Fig. 7. Despite the disturbance, the error signal stably vanishes in about 10 periods, and therefore, the output completely follows the reference without any error. Here our concern is to demonstrate that, without complicated calculations, by choosing PI parameters manually with simple gain adjustments, the suggested HCA strategy works very well and makes the error zero as required. This type of control strategy may be very feasible and useful for industrial control problems as in classical PID control applications. Note in Fig. 6 that, the disturbance is much stronger than the reference level, therefore it takes several periods to make the error vanish. A faster transient response may be achieved by choosing better PI parameters or by employing different control dynamics for the HCA.

The dispersion of the output is shown in Fig. 8. As expected, all harmonic components of the output converge to the exact complex value set by the reference. Here the disturbance input is chosen randomly as a fixed periodic function, however even when the disturbance is a slowly time varying function like in practical applications, the HCA adapts to the disturbance and produces zero steady-state error since for each harmonic component, there is an independent stabilizing PI controller working at a corresponding frequency in the frequency domain.

Despite the given disturbances and the perturbations in the system dynamics, a special and certain control signal at steady state, as shown in Fig. 9, is required to make the system output exactly match to the reference input, and therefore to obtain a zero error. If a different control signal was applied to the system input at steady state, naturally the system output would also change and would not match to the reference input any more. Therefore, an array of controllers is in action and the HCA automatically produces this exact control signal at steady state as required for the compensation.

Let us consider a high constant gain linear controller, which can also make the error small throughout the frequency band, as

$$u(t) = Ke(t). \tag{8}$$

For $K = 50$, the system response is depicted in Fig. 10. The controller performs well; however, although the error is small, it must always remains nonzero because the error is multiplied with a high gain to construct the control signal, which, in turn,

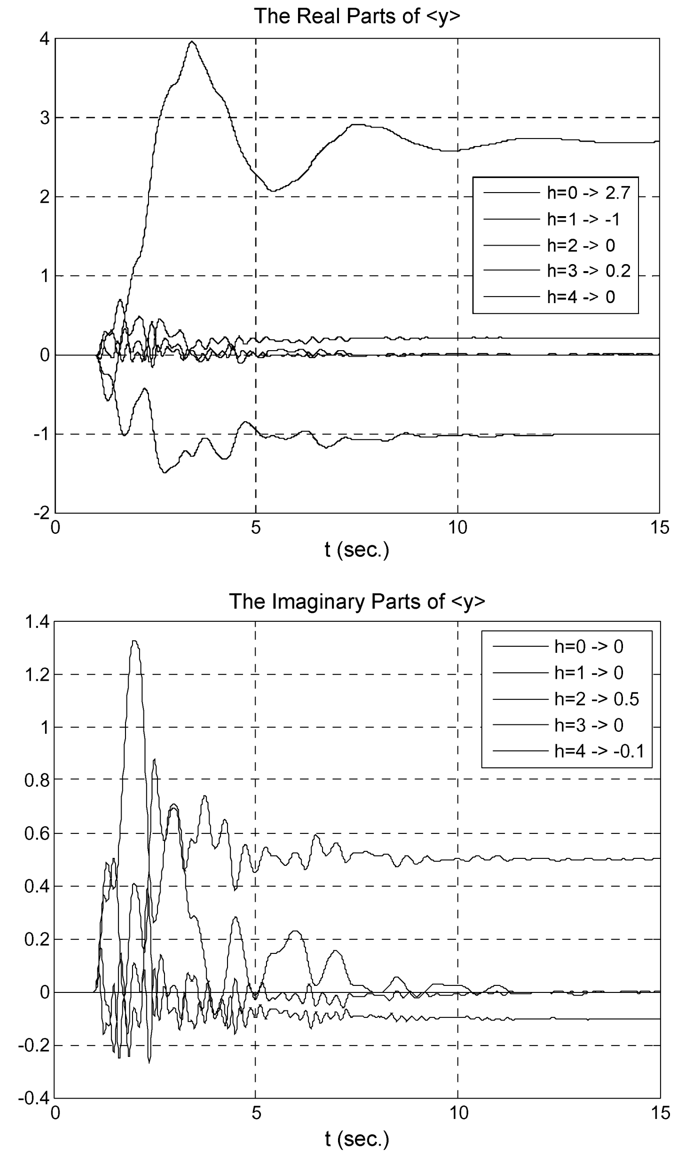

Fig. 8. Real and imaginary parts of $\langle y \rangle$ in controlling with the HCA method for the system without delay.

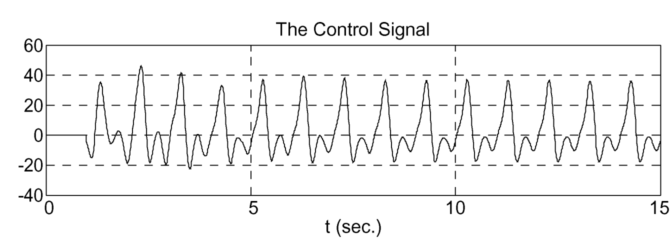

Fig. 9. Control signal produced by the HCA method for the system without delay.

is applied to the system as fast as possible. For higher K values, the system becomes unstable, preventing to lower the error levels for this type of controller. Employing the HCA method, on the other hand, the zero error can be achieved for periodic references or disturbances. This is due to the integral terms working independently for each harmonic component.

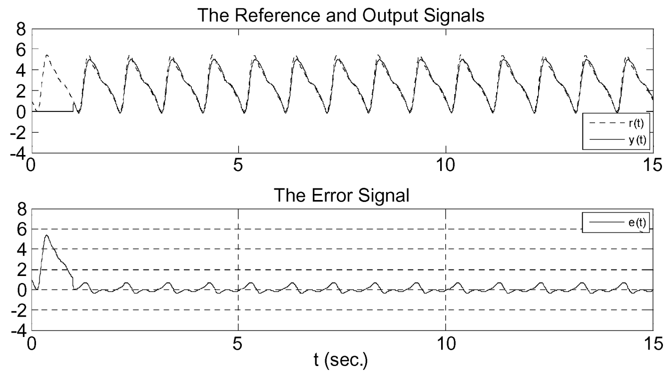

Fig. 10. Output and error in controlling with high gain method for the system without delay.

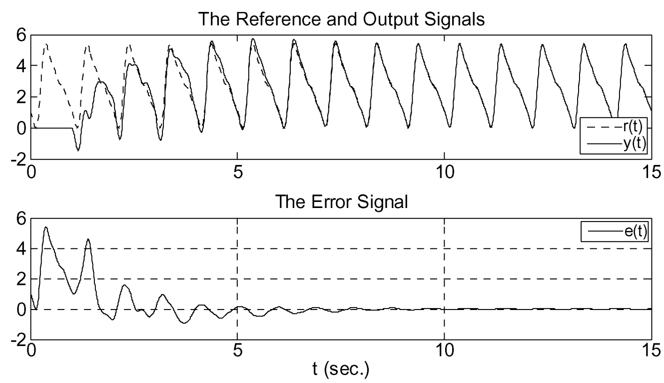

Fig. 11. Output and error in controlling with the HCA method for the system with delay.

B. Behavior of the System With Delay

Let us consider the delay in the system, where $T_d = 0.1$ s. Utilize harmonic PI control array method, and select the PI gains as follows:

$$K_{\rm P} = \mathrm{diag}\begin{pmatrix} 1.1 \\ 0.4+1.4j \\ 0 \\ -8-6j \\ 1.5-4.7j \end{pmatrix}, \qquad K_I = \mathrm{diag}\begin{pmatrix} 0.9 \\ 1.5+4.7j \\ -10+7j \\ -12-9j \\ 5-14j \end{pmatrix}. \tag{9}$$

Here, the following simple manual adjustment strategy is used to obtain the PI gains. First set all PI gains to zero, and starting from the lowest harmonic ($h = 0$), using complex values when needed, manually adjust the corresponding PI gain entries to obtain an acceptable performance at the dispersion graph for the considered harmonic output. At this stage, do not consider the performance for the higher harmonics. Then continue to the next harmonic and set all the gains up to the highest harmonic. If needed, start from the beginning again and continue to adjust the gains to obtain a better control performance. The PI gains above are simply obtained after a few trails and errors as mentioned.

Note that there are complex values in the gain matrices which may be necessary for adjusting the proper magnitudes and phases and obtaining a stable response. The behavior of the system for this case is depicted in Fig. 11, and despite the delay and disturbance, the zero error is still achieved. For comparison, if the behavior of the constant high gain method is considered, at maximum, K can be chosen as 14, and, the results are not at acceptable level for this case. When there is a delay in the system, the high gain control is not useful; however the HCA method can still achieve the zero error in about 10 periods although there is a very strong disturbance.

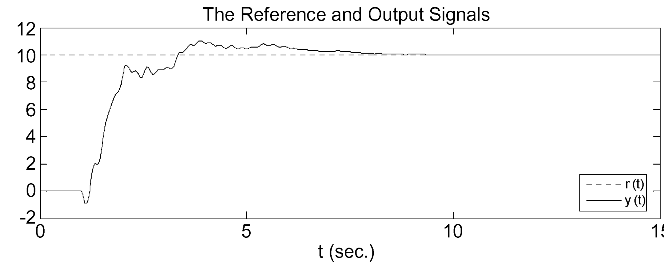

Fig. 12. Output in controlling with the HCA method for a constant reference level.

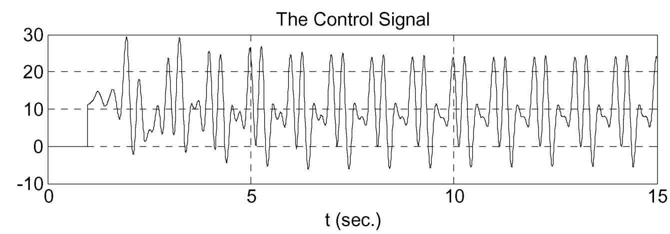

Fig. 13. Control signal produced by the HCA method for a constant reference level.

C. Behavior of the System for a Constant Reference

The HCA method can also be employed for controlling systems with periodic disturbances at a constant reference level. For example, if the output is to be kept at $y = 10$, then the reference dispersion can be selected as

$$\langle r \rangle = [10\,0\,0\,0\,0]^T. \tag{10}$$

Using the same system with delay and the same controller in Section III-B, the result is found as in Fig. 12. Again, despite the delay and the strong disturbance, the output is kept at the reference level. The required control signal produced by the HCA is depicted in Fig. 13.

D. Robustness of HCA

Smith predictor type methods require knowing the system model very precisely, which may be impossible for many practical applications. To test the HCA method for this issue, again consider the same case in Section III-B, but change the system parameter a while keeping the same controller. The results are shown in Fig. 14, and although the pole value is changed by considerable amounts, the zero error performance is still achieved. Therefore, even the dynamical model of the actual system to be controlled is considerably different than the mathematical model used, the HCA method can tolerate it like in robust controllers. However for repetitive control techniques this may introduce serious stability problems.

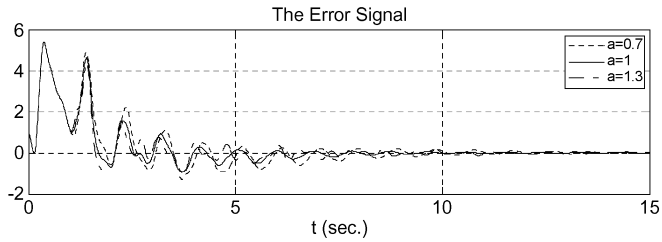

Fig. 14. Error signals in controlling with the HCA method for the system with delay with perturbations in the system parameter.

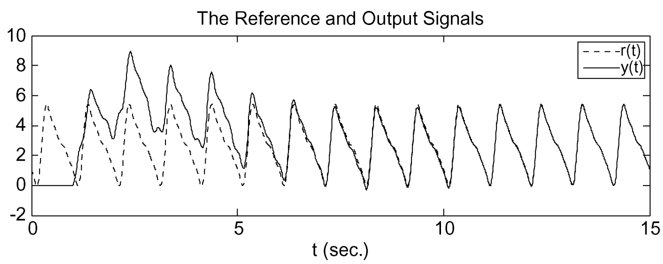

Fig. 15. Output in controlling with the HCA method for the unstable system with delay.

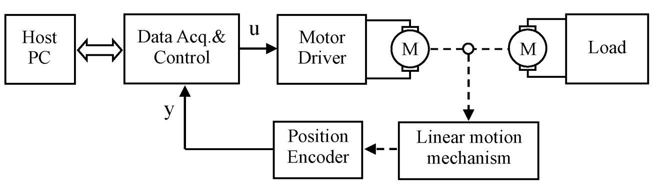

Fig. 16. Designed position control system.

E. Unstable System With Delay

Consider the system in Fig. 5 with a delay and an unstable pole this time, where $a = -1$ and $T_d = 0.1$ s. We can select the PI gains as follows:

$$K_P = \text{diag}(-1.65, 3, 0, 0, 10) \quad K_I = \text{diag}(-0.3, 7, 5, 10, 15). \tag{11}$$

The behavior of the system for this case is depicted in Fig. 15. Despite the unstable system dynamics with the delay and the disturbance, the zero error is easily achieved with the HCA method.

IV. Linear Periodic Position Control Application

An experimental control system is implemented to test and compare the HCA performance. In this aim, a linear periodic position control system is designed. The block diagram of the experimental system is depicted in Fig. 16. Mainly, the system has two permanent magnet dc motors for driving and for loading. The driving motor actuates a linear motion mechanism to control the position. This movement is sensed by a sensor and converted to digital form by an encoder. In order to make a physical loading condition, a loading motor is mechanically attached to the driving motor. The electrical load on the loading motor can be either resistive or nonlinear according to symmetric or asymmetric loading cases. The system has many internal or external nonlinear effects like dead zones and backlash due to the imperfect mechanical structure, which prevents obtaining a simple and fixed mathematical model.

The experimental system basically works as the following: The software, executed by the host PC, controls the system by using an input-output (I/O) interface card. The I/O interface transmits data to the motor driver unit and receives position data from the encoder. The control software, by receiving the position feedback, continuously calculates the necessary control signal and sends it to the interface card for driving the motor unit.

In the experiments, the reference position movement is desired to be in a sinusoidal form with 4 s period. The linear movements in forward and backward directions are chosen close to the limits of ± 10 cm positions. The driving motor supplies a maximum angular speed of 13.5 rad/s in clockwise or anticlockwise direction according to polarity of the input voltage. The motor driver unit provides voltages in the limits of ± 5 V. The system works in real-time with 1 ms sampling period.

In order to evaluate and compare the performance of the HCA method, PI, and internal model control (IMC) methods are also used in the position control system. The methods are tested under no load, symmetric load, and asymmetric load conditions. The asymmetric load is produced by attaching a diode in series with a resistor to the loading motor. In this way, the motor is loaded in only one direction of the linear movement.

In the following subsections, the results obtained using the PI method, the PI plus IMC method, and the HCA method are presented correspondingly. Then the numerical comparisons about these methods are provided. Ensuring that the control signal is always in the given bounds, i.e., preventing the saturation; the actual parameters and coefficients used in the methods are manually adjusted as best as possible in the sense of maximum error suppression for each case.

A. Proportional and Integral Control Method (PI)

In this method the control signal is chosen such that

$$\frac{U(s)}{E(s)} = K_1 + \frac{K_2}{s} \tag{12}$$

where $U(s)$ and $E(s)$ are Laplace transforms of the input and the error signals, K_1 is the proportional gain, and K_2 is the integral gain. Fig. 17 presents the experimental results of the error signals for the three loading conditions. It is seen that although the error is lower with no load condition, for all three cases, the performance is not acceptable. In the asymmetric load condition, the error at the negative alternates is more than at the positive ones. There are some noises on the signals due to the vibrations and the nonlinear effects like dry friction.

B. PI + Internal Model Control Method (PI+IMC)

In this method, a resonant filter which is a second degree marginally stable system function is added to the PI controller considering the sinusoidal reference. The control signal is chosen such that

$$\frac{U(s)}{E(s)} = K_1 + \frac{K_2}{s} + \frac{K_3 s + K_4}{s^2 + w^2} \tag{13}$$

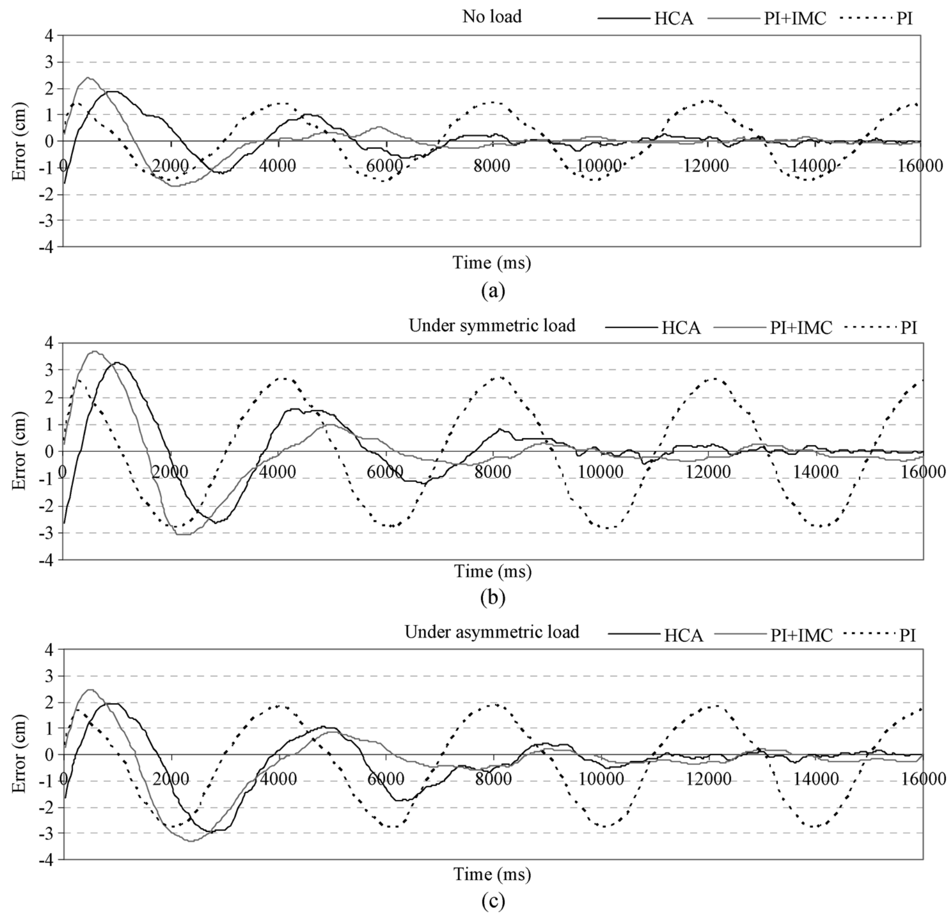

Fig. 17. Comparisons of the error signals for a sinusoidal reference position with an amplitude of 10 cm. (a) Errors with no load, (b) errors with a symmetric load, and (c) errors with an asymmetric load.

where K_1 and K_2 represents the PI gains as before, K_3 and K_4 are the coefficients to completely represent the resonant filter, and, $w = 2\pi/4$ is the angular frequency corresponding to the sinusoidal signal with a period of 4 s. As observed from Fig. 17, this method produces better results than the PI method as expected. The error gradually decreases to acceptable levels for no load case. However for the loading cases, the error cannot be fully diminished to acceptable levels at steady state.

C. HCA Method

The HCA method is applied to the experimental system using 3 harmonics ($H = 3$). It is seen from Fig. 17 that, as in PI+IMC, the HCA method reaches steady state within two period at the no load condition. The control performance is continued at decreasing error levels in the next periods. Under the symmetric or asymmetric loads, the HCA method produces acceptable and gradually decreasing error levels.

Using a Pentium 4 PC running at 3 GHz, and a moderate 12 bit data acquisition card with the sampling frequency of 1 kHz, the implemented HCA algorithm for this case takes a computation time of 0.14 ms of the available 1 ms sampling period. The required computation time can be further significantly reduced by using efficient computation algorithms and dedicated DSP chips.

TABLE I
TOTAL RMS ERRORS IN ONE PERIOD

Method	No load	Symmetric load	Asymmetric load
PI	0.8453	1.4753	1.2684
PI+IMC	0.0716	0.1327	0.1037
HCA (*H*=2)	0.0658	0.1139	0.0863
HCA (*H*=3)	0.0528	0.0867	0.0696
HCA (*H*=4)	0.0434	0.0650	0.0545

D. Comparisons Between the Methods

As observed from Fig. 17, the HCA method performs well in all cases with gradually decreasing error, indicating that the harmonic control values are properly adjusted to compensate the nonlinear effects. Since the reference signal is sinusoidal, the PI+IMC compensates the system at an acceptable level, but the PI controller does not perform well as expected.

In order to compare the error levels at steady state, the root mean square (RMS) error is calculated in the last periods. Table I presents the RMS errors for the PI, the PI+IMC, and the HCA methods. The error rates in the table represent the best results of the collected data for each method. As seen from the results, the HCA method performs more effective than the other methods. Moreover, as H, the control array dimension, increases, the HCA method generates less and less rms error levels.

The structure of the HCA is so convenient that, in general to reduce the steady-state error further, the control array dimension can be easily increased by the addition of two complex numbers to the PI gain matrices. As long as the computation time is allocated in the sampling period, this improvement does not cost anything.

V. Conclusion

A control design method called HCAs is presented. The HCA method offers an effective control solution for systems with periodic references and/or disturbances having a certain number of harmonic components. Its effective compensation and systematic and simple construction may be a useful advantage for industrial applications. Periodic disturbances affect many physical systems and the periodic reference tracking is required for some applications. The HCA method, which is applicable for both cases, can achieve the zero error by automatically producing the control signal composed of the harmonic components at correct magnitudes and phases.

As seen from the simulation examples, despite the delay and a strong disturbance, even with an unstable dynamics, a fast and robust performance can be achieved. The performance of the HCA method is tested and compared using an experimental periodic position control system. It is found that it produces the lowest RMS errors at steady state. As the harmonic array dimension is increased, even lower errors may be achieved gradually.

Since the signals in the harmonic array structure are imaginary/complex valued, it may be impossible to realize the HCA method with an analog circuit. However, using microcontrollers or DSP devices, controllers can be implemented in a flexible way and perform even better than analog ones. Therefore by employing these digital devices and using efficient algorithms, it is quite feasible to realize the HCA algorithm without any problems at even high sampling frequencies. When using the PI method in an HCA structure, the control engineer just need to choose and adjust the complex PI gain coefficients starting from the lower harmonics to the higher ones in an order to achieve the minimum steady state errors and the fast settling times as in the classical PID control.

To further develop this promising method, theoretical investigations about stability and systematic control design should be carried out. Methods may be produced for optimally selecting the PI gain matrices for linear system models. Benefits of using the off diagonal entries in the PI gain matrices, i.e., the cross interactions between the harmonics, can be demonstrated for compensation of nonlinear systems. Other existing control methods can be implemented in the HCA block to achieve even better and faster responses.

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