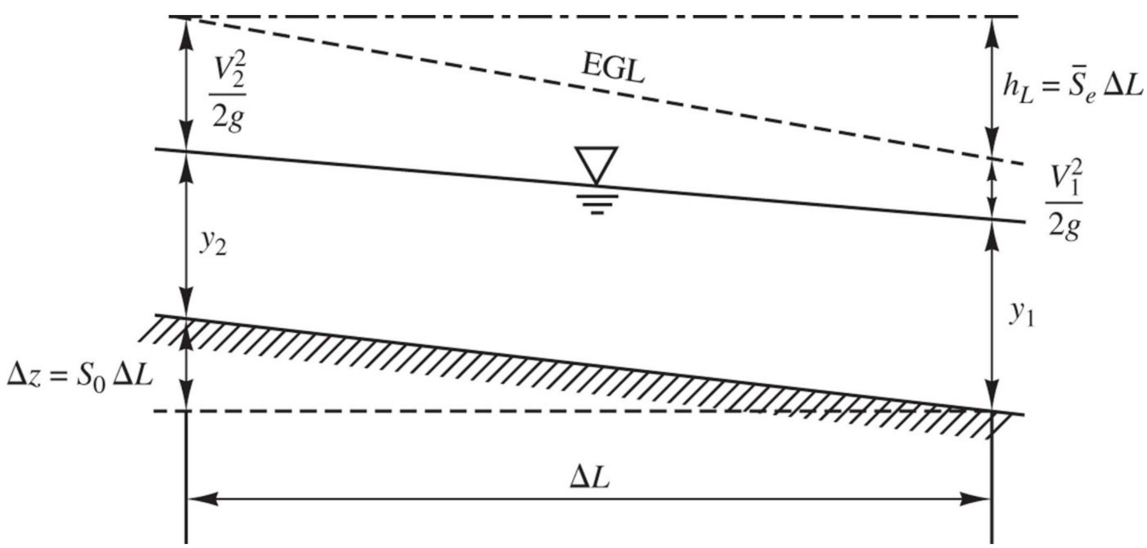
LECTURE 11: Computation of water surface profiles-Standard Step method

Standard step method is directly derived from an energy balance between two neigboring sections 1 and 2, which are seperated by a a sufficiently short distance so that the water surface line can be approximated by a straight line.

Energy balance between 2 neighboring section in the channel:



$$Y_1 + \frac{V^2}{2g} + z_1 = y_2 + z_2 \frac{V^2}{2g} + h_L \quad (11.1)$$

$$y_1 + \frac{V^2}{2g} + z_1 = y_2 + z_2 \frac{V^2}{2g} + \overline{S_e} \Delta L \quad (11.2)$$

$$E_1 = E_2 + \text{losses} \quad (11.3)$$

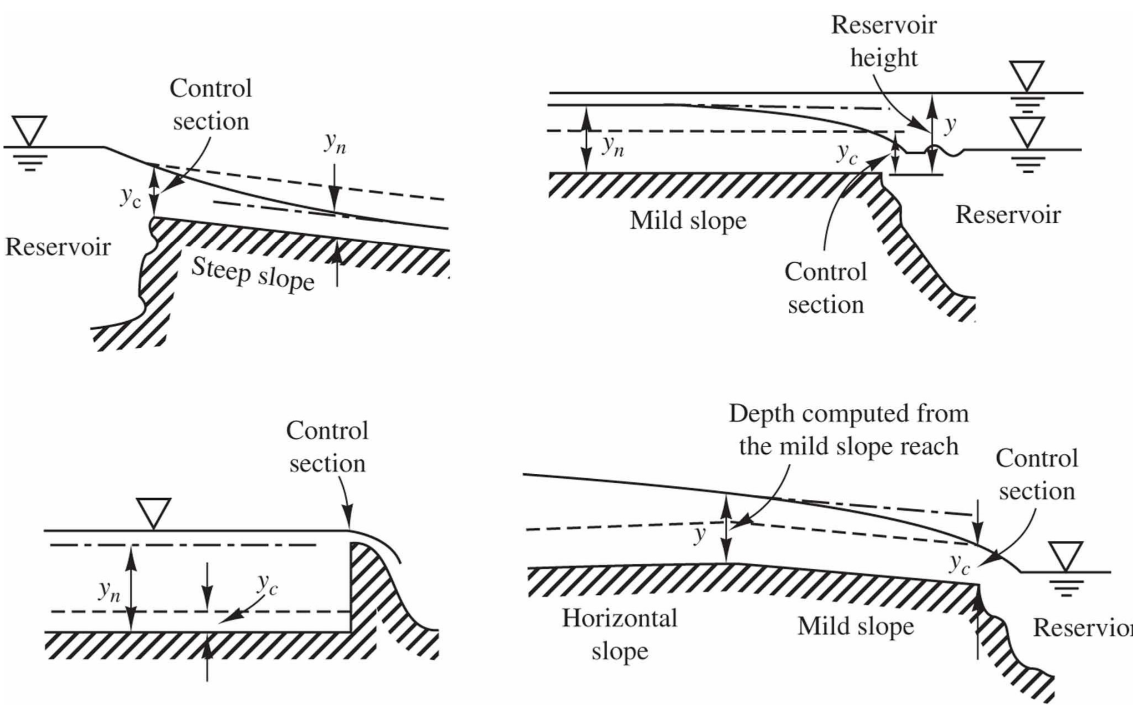
Computation begins at a control section or with a known depth in the channel and proceed upstream or downstream.

A successive computational procedure used to compute the water surface elevation at the next section (upstream or downstream) from the control section. The distance between sections is critical because the water surface will be represented by straight line. Thus, if the depth of flow is changing quickly over short distances, neighboring section should be spaced closed to represent accurately the water surface profile.

The step procedure is carried out in the downstream direction for supercritical flows ($y< yc$) and in the upstream direction for subcritical flows ($y> yc$).

The computation procedure is to determine the depth at a section a distance DL away from a section with a known depth.

Equation 11.2 cannot be solved directly for the unknown depth (e.g. y1) because V1 and $\overline{S_e}$ depend on y1. Therefore, an iterative procedure is required using successive approxiamtions of y1 until the downstream and upstream energy balance is equal to each other. $\overline{S_e}$ is the average of the energy slopes at the upstream and downstream sections.



Control section in open channels

Example 11.1. (Example 6.9, Hwang, 4th Edition)A grouted-riprap, trapezoidal channel (n = 0.0025) with a bottom width of 4 meters and side slopes of m = 1 carries a discharge 12.5 m^3/sec on a 0.001 slope. Compute the backwater curve (upstream water surface profile) created by a low dam that backs water up to a depth of 2 m immediately behind the dam. Specifically, water depths are required at critical diversion points that are located at distances of 188 m, 423 m, 748 m, and 1,675 m upstream of the dam.

Solution:

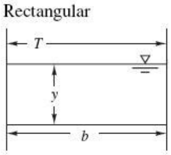
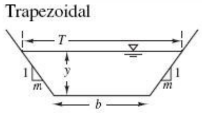
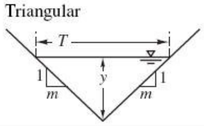
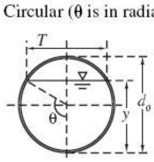
1. Calculate normal depth,

$$S_o = S_e \text{ for uniform flow conditions}$$

$$\frac{Q}{A} = \frac{1}{n} \, . \, {R_h}^{2/3} . {S_o}^{1/2}$$

Use the table given below to calculate area, R, P etc. for trapezoidal channel;

TABLE 6.1 Cross-Sectional Relationships for Open-Channel Flow

Section Type	Area (A)	Wetted perimeter (P)	Hydraulic Radius (R_h)	Top Width (T)	Hydraulic Depth (D)
Rectangular 	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y
Trapezoidal 	$(b + my)y$	$b + 2y\sqrt{1 + m^2}$	$\frac{(b + my)y}{b + 2y\sqrt{1 + m^2}}$	$b + 2my$	$\frac{(b + my)y}{b + 2my}$
Triangular 	my^2	$2y\sqrt{1 + m^2}$	$\frac{my}{2\sqrt{1 + m^2}}$	$2my$	$\frac{y}{2}$
Circular (θ is in radians) 	$\frac{1}{8}(2\theta - \sin 2\theta)d_0^2$	θd_0	$\frac{1}{4}(1 - \frac{\sin 2\theta}{2\theta})d_0$	$(\sin \theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{2\theta - \sin 2\theta}{\sin \theta}\right)d_0$

Source: V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

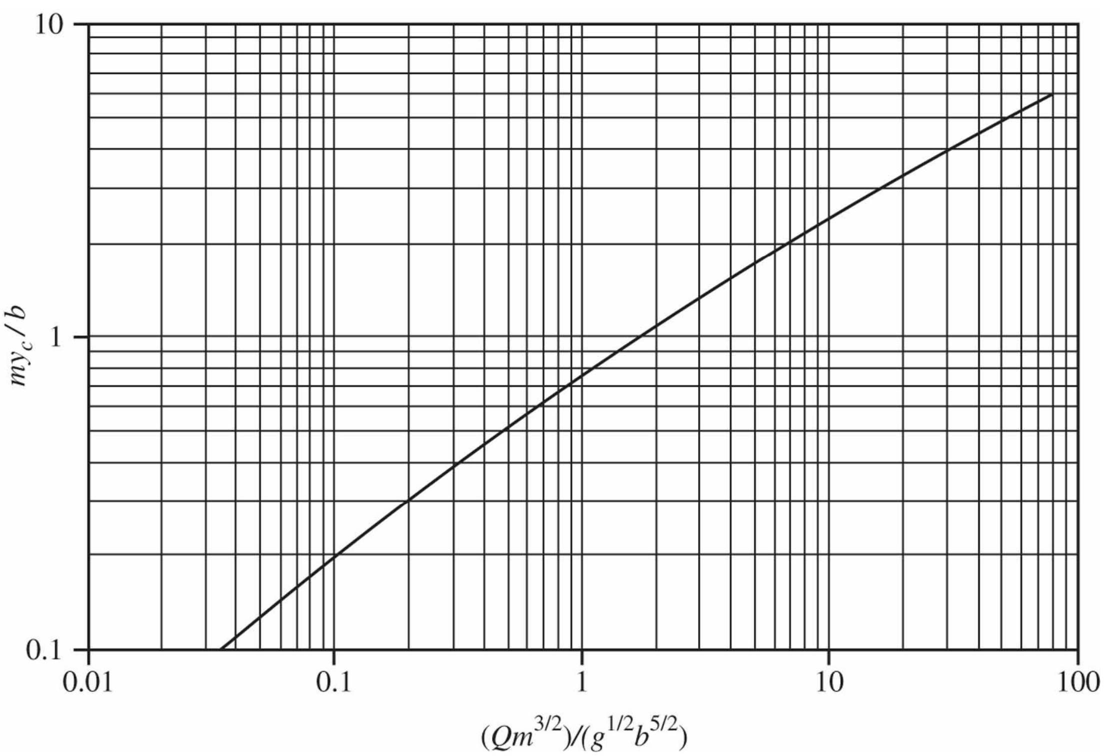
$$\frac{12.5}{(b+m.y).y} = \frac{1}{0.025} . \left(\frac{(b+m.y).y}{b+2.y.\sqrt{1+m^2}}\right)^{2/3} . (0.001)^{1/2}$$

$$\frac{12.5}{(1.y+4.m).y} = \frac{1}{0.025} . \left(\frac{(4+y).y}{4+2.y.\sqrt{1+1}}\right)^{2/3} . (0.001)^{1/2}$$

By trial and error ; $y_n = 1.66$ m

2. Calculate critical depth

(a)



$$\frac{Q.m^{3/2}}{g^{1/2}.b^{5/2}} = \frac{(12.5).(1)^{3/2}}{(9.81)^{1/2}.(4)^{5/2}} = 0.125$$

$$\frac{m.y_c}{b} = 0.230$$

$$y_c = \frac{(0.23).(4\ m)}{1}$$

$$y_c = 0.92\ m$$

Known water depth ; $y = 2$ m

Normal depth; $y_n = 1.66$ m

Critical depth; $y_c = 0.92$ m

From Table 6.3, $y > y_n$ & $y > y_c$ ➔ y>yn>yc ➔ MILD SLOPE ➔ M-1 CURVE

TABLE 6.3 Characteristics of Water Surface Profile Curves

Channel	Symbol	Type	Slope	Depth	Curve
Mild	M	1	$S_0 > 0$	$y > y_n > y_c$	M-1
Mild	M	2	$S_0 > 0$	$y_n > y > y_c$	M-2
Mild	M	3	$S_0 > 0$	$y_n > y_c > y$	M-3
Critical	C	1	$S_0 > 0$	$y > y_n = y_c$	C-1
Critical	C	3	$S_0 > 0$	$y_n = y_c > y$	C-3
Steep	S	1	$S_0 > 0$	$y > y_c > y_n$	S-1
Steep	S	2	$S_0 > 0$	$y_c > y > y_n$	S-2
Steep	S	3	$S_0 > 0$	$y_c > y_n > y$	S-3
Horizontal	H	2	$S_0 = 0$	$y > y_c$	H-2
Horizontal	H	3	$S_0 = 0$	$y_c > y$	H-3
Adverse	A	2	$S_0 < 0$	$y > y_c$	A-2
Adverse	A	3	$S_0 < 0$	$y_c > y$	A-3

The depth (y= 2 m) just upstream from the dam is the control section designated as section 1. Energy balance computations begin here and progress upstream (backwater) because the flow is subcritical (yc < yn).

Since the profile has an M-1 classification, the flow depth will approach normal depth asymptotically as the computations progress upstream.

Since y/yc >1 and y/yn >1; the value dy/dx is positive, indicating that water depth increases in the direction of flow.

The results of the calculations are given in following pages.

Table 6.4 Water Surface Profile (Backwater) Computations Using the Standard Step Method (Example 6.9)

TABLE 6.4 (a) Water Surface Profile (Backwater) Computations Using the Standard Step Method (Example 6.9)

(1) Section	(2) U/D	(3) y (m)	(4) z (m)	(5) A (m^2)	(6) V (m/sec)	(7) $V^2/2g$ (m)	(8) P (m)	(9) R_h (m)	(10) S_e	(11) $S_{e(avg)}$	(12) h_L (m)	(13) Total Energy (m)
1	*D*	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000538	0.1011	2.156
2	*U*	1.94	0.188	11.52	1.085	0.0600	9.487	1.215	0.000567	(ΔL = 188 m)		2.188
Note: The trial depth of 1.94 m is too high; the energy does not balance. Try a lower upstream depth.												
1	*D*	2.00	0.000	12.00	1.042	0.0553	9.657	1.243	0.000508	0.000554	0.1042	2.159
2	*U*	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000600	(ΔL = 188 m)		2.160
Note: The trial depth of 1.91 m is correct. Now balance energy between sections 2 and 3.												
2	*D*	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000673	0.1582	2.319
3	*U*	1.80	0.423	10.44	1.197	0.0731	9.091	1.148	0.000745	(ΔL = 235 m)		2.296
Note: The trial depth of 1.80 m is too low; the energy does not balance. Try a higher upstream depth.												
2	*D*	1.91	0.188	11.29	1.107	0.0625	9.402	1.201	0.000601	0.000659	0.1549	2.315
3	*U*	1.82	0.423	10.59	1.180	0.0710	9.148	1.158	0.000716	(ΔL = 235 m)		2.314
Note: The trial depth of 1.82 m is correct. Now balance energy between sections 3 and 4.												

Column (1) Section numbers are arbitrarily designated from downstream to upstream.

Column (2) Sections are designated as either downstream (*D*) or upstream (*U*) to assist in the energy balance.

Column (3) Depth of flow (meters) is known at section 1 and assumed at section 2. Once the energies balance, the depth is now known at section 2, and the depth at section 3 is assumed until the energies at sections 2 and 3 balance.

Column (4) The channel bottom elevation (meters) above some datum (e.g., mean sea level) is given. In this case, the datum is taken as the channel bottom at section 1. The bottom slope and distance interval are used to determine subsequent bottom elevations.

Column (5) Water cross-sectional area (square meters) corresponds to the depth in the trapezoidal cross section.

Column (6) Mean velocity (meters per second) is obtained by dividing the discharge by the area in column 5.

Table 6.4 (continued) Water Surface Profile (Backwater) Computations Using the Standard Step Method (Example 6.9)

Column (7) Velocity head (meters).
Column (8) Wetted perimeter (meters) of the trapezoidal cross section based on the depth of flow.
Column (9) Hydraulic radius (meters) equal to the area in column 5 divided by the wetted perimeter in column 8.
Column (10) Energy slope obtained from Manning equation (Equation 6.27a).
Column (11) Average energy grade line slope of the two sections being balanced.
Column (12) Energy loss (meters) from friction between the two sections found using $h_L = S_{e(avg)}(\Delta L)$ from Equation 6.26b.
Column (13) Total energy (meters) must balance in adjacent sections (Equation 6.26b). Energy losses are always added to the downstream section. Also, the energy balance must be very close before proceeding to the next pair of adjacent sections or errors will accumulate in succeeding computations. Thus, even though depths were only required to the nearest 0.01 m, energy heads were calculated to the nearest 0.001 m.

TABLE 6.4 (b) Water Surface Profile (Backwater) Computations Using the Direct Step Method (Example 6.9)

Section	*U/D*	*y* (m)	*A* (m^2)	*P* (m)	R_h (m)	*V* (m/sec)	$V^2/2g$ (m)	*E* (m)	S_e	ΔL(m)	Distance to Dam (m)
1	*D*	2.00	12.00	9.657	1.243	1.042	0.0553	2.0553	0.000508		0
2	*U*	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601	186	186
	A distance of 186 m separates the two flow depths (2.00 m and 1.91 m).										
2	*D*	1.91	11.29	9.402	1.201	1.107	0.0625	1.9725	0.000601		186
3	*U*	1.82	10.59	9.148	1.158	1.180	0.0710	1.8910	0.000716	239	425
	A distance of 239 m separates the two flow depths (1.91 m and 1.82 m).										