Abstract

This paper deals with the analysis and proper design of parabolic reflector antennas to obtain pencil beam, cosecant-squared and inverse cosecant-squared radiation patterns for air and coastal surveillance radars. The analytical regularization method (ARM) is used to solve the problem of E-polarized wave diffraction by parabolic shaped perfectly electrical conductive (PEC) cylindrical reflector with finite thickness.

1. Introduction

The parabolic reflectors are one of the most popular antennas commonly used in microwave radar, power transmission, satellite and point-to-point communication systems [1-3]. They have generally large physical dimensions with respect to the wavelength. Thus, high frequency electromagnetic wave scattering techniques such as geometrical optics (GO), physical optics (PO), aperture integration (AI) and geometrical theory of diffraction (GTD) are mostly used for determining the far field antenna characteristics [4]. Some direct numerical techniques; i.e. method of moments (MoM), finite element method (FEM) and finite difference methods, can be employed especially for non-canonical structures [5]. However, in many cases, the complexity of some cavity or aperture geometries creates hard numerical convergence problems because of the computational instabilities. The origin of these problems is related to the nature of the direct numerical methods, which reduce a diffraction boundary value problem (BVP) to the functional equation of the first kind. A typical first kind algebraic equation system frequently has a singular kernel and a very big condition number value that can cause unstable numerical process. Thus, minimizing the computational error by increasing the truncation number of the algebraic equation set cannot be guaranteed [6-7].

To obtain pencil beam and cosecant-squared radiation patterns, the reflector geometry is cut and bended partially, and the effects on the radiation characteristics are analyzed parametrically. Analytical regularization method (ARM), which is implemented for solving the 2D problem of E-polarized wave diffraction by arbitrary shaped, smooth and perfectly conductive cylindrical obstacles, is applied to obtain fast, accurate and reliable results for the parabolic reflector structure (see Fig. 2). The ARM equivalently reduces the original
diffraction BVP to the algebraic system of the second kind of the form \((I+H)x=b\), \(x,b \in l_2\), where \(I\) and \(H\) are correspondently identical and compact operators in space \(l_2\) of summable sequences \[8\]. Therefore, the numerical stability of solving process is guaranteed for arbitrary big truncated matrix to enable us to reach the solution of original BVP with any required accuracy \[9\].

In this study, new studies on switching different beam types (pencil, cosec², inverse cosec²) are also introduced just by changing flare angles of the feed horn. It is shown that such major pattern switching can be reached by using asymmetric illumination. By this way, same reflector structure can be used for both air and coastal surveillance radars.

2. ARM Formulation

Scalar diffraction problem of an infinitely long, smooth, longitudinally homogeneous and perfectly conducting cylindrical obstacle corresponds to the Dirichlet boundary condition for E-polarized incident wave. Consider its XOY plane cross section is denoted by the closed contour \(S\), the incident and scattered scalar wave functions \((u'(p)\) and \(u''(p))\) must satisfy the following Helmholtz equation given in Eq. (1) and the Dirichlet boundary condition in Eq.(2), also with the Sommerfeld radiation condition.

\[
(\Delta+k^2)u'(p)=0, \quad p \in \mathbb{R}^2\backslash S
\]

\[
u''+(p)=u''-(p)=u'(p), \quad p \in S
\]

where, \(S\) smooth contour of the domain \(D\) in 2D space \(\mathbb{R}^2\) that belongs to the smoothness class \(C^2\). \(u''(p)\) and \(u'(p)\) are limiting values of \(u'(p)\) in the inner and the outer sides of \(S\), respectively. The solution of the BVP is written in Eq. (3) using Green’s formula and the boundary condition in Eq. (2).

\[
\int_{\partial S} [H^1(k'|q-p)|\nabla u'(q)]d\rho = -u'(q), \quad q,p \in S
\]

where, \(Z(p)=\frac{\partial u''(p)}{\partial n}-\frac{\partial u''(p)}{\partial n}, \quad p \in S; \quad n\) is the unit outward with respect to \(S\) normal of the point \(p\). The unknown function \(Z(p)\) is constructed by solving Eq. (3), and using parameterization of the \(S\) contour specified by the function \(\eta(\theta)=(x(\theta),y(\theta))\) that smoothly parameterizes the contour \(S\) by the points of \(\theta \in [-\pi,\pi]\). The integral equation representation of the first kind in Eq. (3) can be equivalently rewritten by means of the \(\eta(\theta)\) parameterization as follows:

\[
\int_{\pi}^{\pi} \frac{1}{2\pi} \ln\left|2\sin\frac{\theta-\tau}{2}\right| K(\theta,\tau)Z_D(\tau)d\tau = g(\theta), \quad \theta \in [-\pi,\pi]
\]

with the unknown function \(Z_D(\tau)\) and the given function \(g(\theta)\), where

\[
Z_D(\theta)=l(\theta)Z(\eta(\theta)), \quad g(\theta)=-u'(\eta(\theta)), \quad \theta \in [-\pi,\pi]
\]

\[
l(\theta)=[(x(\theta))^{12}+[y(\theta))]^{1/2}>0, \quad x(\theta),y(\theta) \in C^-(Q')
\]

Here \(K(\theta,\tau)\) function is rather smooth section of the Green’s function in comparison with \(\ln\left|2\sin\frac{\theta-\tau}{2}\right|\) part that represents the main singularity of the Eq. (4) \[9\]. The functions in Eq. (4) are represented by their Fourier series expansions with \(k_{s,m}, m, g_m\) coefficients. Subsequently, one can obtain an infinite system of the linear algebraic equations of the second kind:

\[
\hat{z}_s + \sum_{s,m} \hat{k}_{s,m} \hat{z}_m = \hat{g}_s, \quad s=\pm1,\pm2,\ldots
\]
where, \( \hat{k}_{r,m} = -2\tau_r \delta_1 \left[ k_{r,m} + \frac{1}{2} \delta_{s,0} \delta_{m,0} \right] \), \( \hat{z}_r = r^{-1} \delta_{r,0} \), \( g = -2\tau_r \delta_{r,0} \), \( \tau_r = \max(1,|n|^{1/2}) \), \( n = 0, \pm 1, \pm 2, \ldots \) (8)
and \( \delta_{s,0} \) is the Kronecker delta function. Finally, the scattered field \( u'(q) \) for \( q \in \mathbb{R}^2 \) are obtained by the integral equation representation of the Eq. (4) with any required accuracy by the truncation method.

3. Geometrical Design and Numerical Results

![Figure 1](image1.png)

(a) Comparisons of the ARM calculations with analytical results
   a) Uniform plane wave illumination, circular cylinder, normalized radius \((ka)=3.1\)
   b) Near field analysis of the feed horn antenna

The ARM procedure is verified by the analytical solution for induced current density distribution function of the infinitely long circular cylinder for the oblique incident plane (see Fig. 1a) and also by the analytical solution of radiation from open-ended waveguide [10-11]. The near field analysis of the feed horn model with \(3\lambda\) flare length is presented in Fig. 1b.

The 2D configuration of the parabolic reflector and its feed horn antenna are shown in Fig. 2. To obtain pencil beam, the parabolic reflector antenna indicated with dashed line is used. To obtain cosecant-squared radiation pattern, the bottom side of the reflector antenna is partially cut and bended by 5, 10, 15 and 20 degrees, respectively. The performance results of pencil beam and cosecant-squared radiation patterns are given in Fig. 3.

![Figure 2](image2.png)

Figure 2 XOY-plane cross-section geometry of parabolic reflector and feed horn
4. Conclusion

This paper has presented the geometric analysis of the radiation characteristics of parabolic reflector antenna illuminated by H-plane horn feeder. The analytical regularization method is used to perform 2-D analysis of E-polarized electromagnetic wave scattering problem as a fast, accurate and stable numerical technique. The ARM algorithm is firstly compared with analytical data for verification. The results of different geometrical configurations such as, symmetric, offset cut, and bended reflector shapes are demonstrated on the radiation patterns to attain pencil and fan beams, which are required for microwave radar systems used for air and coastal surveillance.

References