Calculus I: Topics covered in the class.

From Thomas' Calculus Early Transcendentals 13th edition.

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Please let me know of any mistakes in these notes.

Contents

Week 1

1.1 Functions and Their Graphs

1. Definition of **function**, **domain** and **range**.

2. The domain and range of a function can be any sets of objects, but in this class, they will always be the set of real numbers $\mathbb{R}$. Also we will never consider complex numbers in this course.

3. If the domain of the function is not specified then the convention is to take its domain as the largest set of real x-values for which the rule of the function is defined.

4. Example 1.

5. Graphs of functions.

6. Vertical line test for a function.

7. Piecewise-Defined Functions. Important example: Absolute value function.

8. Increasing and decreasing functions.

9. Even and odd functions and their symmetry properties.

10. Common functions: linear functions $f(x) = mx + b$, power functions $f(x) = x^a$ where a is constant real number, polynomials.

11. Exercises 1.1: 1-8, 15-22, 25-28, 37-58

1.2 Combining Functions; Shifting and Scaling Graphs

1. Sums, differences, products and quotients. Example 1.

2. Composite functions. Example 2.

3. Shifting the graph of a function: vertical and horizontal shifts. Example 3.

4. Vertical and Horizontal Scaling and Reflecting Formulas. Example 4.

5. Exercises 1.2: 1, 3, 5, 9, 13aef, 15ef, 17, 19, 21, 23, 27, 33, 37, 39, 41, 45, 47, 51, 67, 71, 77.

Week 2

1.3 Trigonometric Functions

1. Angles.

2. The six basic trigonometric functions.

3. Periodicity and graphs of the trigonometric functions.

4. Trigonometric identities: addition formulas, double angle formulas, half angle formulas, the law of cosines.

5. Two Special Inequalities.

6. Exercises 1.3: 5, 7, 11, 13, 15, 17, 19, 31, 41, 43, 49, 51, 53, 55, 59

1.5 Exponential Functions

1. Definition of a^x when x is an integer, a rational number and an irrational number (such as $2^{\sqrt{3}}$).

2. Rules of exponents. Example 2.

3. The natural exponential function e^x.

4. Exercises 1.4: 1, 7, 9, 11, 15, 19, 21, 23

1.6 Inverse Functions and Logarithms

1. One-to-one functions. Example 1.
2. The horizontal line test for one-to-one functions.
3. Inverse functions. Example 2.
4. Finding inverses: The graph of f^{-1}. Example 3-4.
5. Logaritmic functions.
6. Properties of logarithms. Theorem 1. Example 5.
7. Inverse properties for a^x and $\log_a x$.
8. Definition of $a^x = e^{x \ln a}$.
9. Change of base formula.
10. Inverse tirgonometric functions.
11. The arcsine and arcosine functions. Example 8.
12. Identities involving arcsine and arcosine
13. Exercises 1.6: 7, 11, 13, 19, 31, 33, 41aedf, 43, 47, 53, 57, 63ab, 67, 69, 75a, 77ce

Week 3

2.1 Rates of Change and Tangents to Curves

1. Average and instantaneous speed. Example 1-2.
2. Average rates of change and secant lines.

2.2 Limit of a Function and Limit Laws

1. Limit of function values. Example 1-4
2. Limit laws. Theorem 1. Example 5. Theorem 2-3. Example 6.
3. Eliminating common factors from zero denominators.Example 7.
4. The sandwich theorem. Example 10.
5. Exercises 2.2: 1, 3, 5, 15, 22, 23, 27, 33, 35, 41, 47, 53, 65, 67a, 77, 79

2.4 One Sided Limits

1. Approaching a Limit from One Side. Theorem 6.
2. Limits involving $\sin\theta\theta$. Theorem 7. Example 5-6.
3. Exercises: 1, 5, 17, 21, 27, 33, 39

Week 4

2.5 Continuity

1. Continuity at a Point. Example 1, 2, 3. Continuity test.
2. Continuous functions. Theorem 8. Example 6, 7.
3. Theorem 9. Example 8. Theorem 10.
4. Intermediate value theorem for continuous functions. Example 10
5. Exercises 2.5: 5, 29, 30, 33, 43, 47, 55, 56.

2.6 Limits Involving Infinity; Asymptotes of Graphs

1. Finite limits as $x \to \pm\infty$. Theorem 12. Example 2
2. Limits at infinity of rational functions. Example 3.
3. Horizontal asymptotes. Example 4, 6, 7, 8, 9.
4. Infinite limits. Example 11, 13, 14.
5. Vertical asymptotes. Example 16, 17, 18, 19.
6. Exercises 2.6: 1, 9, 11, 17, 19, 21, 23, 27, 33, 43, 53, 59, 81, 83,

Week 5

3.1 Tangents and the Derivative at a Point

1. Finding a tangent to the graph of a function. Example 1.
2. Definition of derivative of a function at a point.
3. Exercises 3.1: 11, 13, 35, 36.

3.2 The Derivative as a Function

1. Alternative formula for the derivative.
2. Calculating derivatives from the definition. Example 1, 2
3. Notations.
4. When does a function not have a derivative at a point?
5. Differentiable functions are continuous. Theorem 1 and its proof.
6. Exercises 3.2: 13, 17, 53, 58

3.3 Differentiation Rules

1. Powers, Multiples, Sums, and Differences. Derivative of a Constant Function, Derivative of a Positive Integer Power, Power Rule (General Version), Example 1.
2. Derivative Sum Rule and proof. Example 3, 4.
3. Derivatives of Exponential Functions. Example 5.
4. Derivative of Product and Quotient Rule. Example 6, 7, 8.
5. Second and higher order derivatives. Example 10
6. Exercises 3.3: 1, 5, 7, 15, 23, 37, 39, 47, 53, 55, 59, 67, 69.

Week 6: Cumhuriyet Bayramı. No Class.

Week 7

3.5 Derivatives of Trigonometric Functions

1. Derivative of the Sine Function. Example 1.
2. Derivative of the Cosine Function. Example 2.
3. Derivatives of the Other Basic Trigonometric Functions. Example 5, 6.
4. Exercises 3.5: 5, 13, 17, 31, 57, 49, 51, 53, 57.

3.6. The Chain Rule

1. Derivative of a Composite Function. Example 1, 3, 4.
2. Repeated Use of the Chain Rule. Example 5.
3. The chain rule with powers of a function. Example 6, 7, 8.
4. Exercises 3.6: 13, 15, 39, 57, 67, 83.

3.7 Implicit Differentiation

1. Implicitly Defined Functions. Example 3.
2. Derivatives of Higher Order. Example 4, 5.
3. Exercises 3.7: 1, 11, 13, 23, 33, 37, 41, 47.

3.8 Derivatives of Inverse Functions and Logarithms

1. Derivatives of Inverses of Differentiable Functions. Theorem 3. Example 2.
2. Derivative of the Natural Logarithm Function. Example 3, 4.
3. The derivatives of a^u and $\log_a u$. Example 5.
4. Logarithmic differentiation.
5. Irrational exponents and the power rule (general version). Example 7.
6. Theorem 4. The number e as a limit.
7. Exercises 3.8: 7, 19, 33, 51, 61, 69, 81, 89, 93

Week 8: Midterm Week. No Class

Week 9

3.9 Inverse Trigonometric Functions

1. Inverses of $\tan x$, $\sec x$.
2. The derivative of $y = \sin^{-1} u$. Example 2.
3. The derivative of $y = \tan^{-1} u$.
4. Exercises 3.9: 9, 11, 21, 23, 33, 41, 52.

3.10 Related Rates

1. 4.1 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 3.10: 9, 20, 21, 31, 37

4.5 Indeterminate Forms and L'Hôpital's Rule

1. 4.2. from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 4.5: 5, 13, 23, 29, 41, 45, 47, 51, 53, 61, 67, 79

Week 10

4.1 Extreme Values of Functions

1. 4.3 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 4.1: 55, 61, 63, 65, 71.

4.2 The Mean Value Theorem

1. 2.7 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 4.2: 23, 27, 63, 65.

4.3 Monotonic Functions and the First Derivative Test

1. Increasing Functions and Decreasing Functions. Corollary 3. Example 1
2. First derivative test for local extrema. Example 2, 3.
3. Exercises 4.3: 33, 35, 41, 43, 51, 55, 67, 69, 74, 77.

4.4 Concavity and Curve Sketching

1. Concavity. The Second Derivative Test for Concavity. Example 1, 2.
2. Points of Inflection. Example 3, 4, 5.
3. Second Derivative Test for Local Extrema. Example 7, 8, 9, 10
4. Exercises 4.4: 17, 33, 37, 41, 49, 87, 93, 105, 121

Week 11

4.6 Applied Optimization

1. 4.6 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 4.6: 1, 3, 5, 7, 9, 19, 34, 52, 58

3.11 Linearization and Differentials

1. 4.7 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 3.11: 11, 13, 15, 17.

5.3 The Definite Integral

1. 5.1 from http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf.
2. Exercises 5.3: 17, 21.

Week 12

From http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf:

1. 5.2 The fundamental theorem of calculus.
2. 5.3 The Method of Substitution
3. 5.4 Areas of Plane Regions
4. 5.5 Integration by Parts

Week 13

From http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf:

1. 5.6 Integrals of Rational Functions
2. 5.7 Inverse Substitutions
3. 5.8 Indefinite Integrals Exercises
4. 5.9 Improper Integrals

Week 14-15

From http://mimoza.marmara.edu.tr/~taylan.sengul/files/calc1-main.pdf:

1. 6.1 Volumes Using Cross-Sections
2. 6.2 VolumesUsingCylindricalShells
3. 6.3 Arc Length
4. 6.4 Areas of Surfaces of Revolution
5. 5.10 Trigonometric Integrals